16.

THE 2-FACTOR DESIGN

The model has two factors A: $A_1, \ldots, A_a$
B: $B_1, \ldots, B_b$
which determines the linear model

$$\boxed{E[y] = \beta_{11} x_{11} + \beta_{21} x_{21} + \ \ldots \ + \beta_{ab} x_{ab}}$$

Where $x_{ij} = 1$ if treatment A_iB_j is applied and zero otherwise.
If we apply A_iB_j n_{ij} times we obtain $E[y] = X\beta$

$$X = \begin{vmatrix} 1 & & 0 & & 0 & \\ 1 & & . & & . & \\ 1 & n_{11} & 0 & & . & \\ 0 & & 1 & n_{ij} & 0 & \\ & & 1 & & 0 & \\ 0 & & . & & 1 & \\ 0 & & 0 & & 1 & n_{ab} \\ 0 & & 0 & & 1 & \end{vmatrix}$$

and

$$y = \begin{bmatrix} n_1 & obs & from & A_1 & B_1 \\ n_2 & " & " & A_2 & B_2 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ n_a & " & " & A_a & B_b \end{bmatrix}$$

The least squares estimator is given by $b = (X'X)^{-1} \ X'y$

$$where\,(X^{'}X)^{-1} = \begin{bmatrix} \frac{1}{n} & & & 0 \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ 0 & & & \frac{1}{n} \end{bmatrix}$$

and $X^{'}y = \begin{bmatrix} T_{11} \\ T_{21} \\ T_{ab} \end{bmatrix}$

The least squares estimators of the β are

$$b = \begin{bmatrix} \frac{1}{n} & & & 0 \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ 0 & & & \frac{1}{n} \end{bmatrix} \begin{bmatrix} T_{11} \\ T_{21} \\ T_{ab} \end{bmatrix} = \begin{bmatrix} \frac{T_{11}}{n} \\ \cdot \\ \cdot \\ \cdot \\ \frac{T_{ab}}{n} \end{bmatrix} = \begin{bmatrix} \bar{y}_{11} \\ \cdot \\ \cdot \\ \cdot \\ \bar{y}_{ab} \end{bmatrix}$$

where T_{ij} is the total of the observations from treatment A_i B_j and $\bar{y}_{ij}$ is the average. In other words, the average of the treatment gives the least squares estimator for this design.

16.1 TEST FOR INTERACATION AND TREATMENT EFFECTS

Let $C_A = (C_{11}\ \ C_{21}\ \ ...\ C_{a1})$ and $C_B = (C_{12}\ \ C_{22}\ \ ...\ C_{b2})$
be orthogonal matrices for A and B respectively.

Then $C = C_A \otimes C_B$
$= (C_{11} \otimes C_{12}\ \ \ C_{11} \otimes\ C_{22}\ \ \ ...\ \ \ C_{a1} \otimes C_{b2})$

is an orthogonal contrast matrix where
$\alpha_{ij} = (C_{i1} \otimes C_{j2})\,\beta$
$= C\,\beta$

Remember that the α_{ij} for $i \neq 1$, $j \neq 1$ are the interaction contrasts.
We then test the hypothesis of no interaction.
If we find no evidence against the null hypothesis of no interaction(s) we then test for effects due to A and B.
We have the following table:

Source	DF
Model Mean	1
Difference Model fitting A - Mean Model	(a-1)
Difference Model fitting B - Mean Model	(b-1)
Difference Model fitting AB - Model [(A-1) + (B-1) + 1]	(a-1)(b-1)
Residual	N - ab
Total	N where N = abn

Rule: Axiom: We always speak of the balanced case ie.

$$n_{11} = n_{12} = \ldots = n_{ab} = n$$

The sum of squares of ($C_{i1} \otimes C_{j2}$)is na_{ij^2} where $na_{11}{}^2 = \dfrac{G^2}{nab}$

$$na_{i1^2} = \frac{1}{ab}[C_{i1} \begin{matrix} T_1 \\ . \\ . \\ T_a \end{matrix}] \; whew \; T_i = \sum_{j=1}^{b} T_{ij}$$

The SS of the effect of A is thus

$$\sum na_{i1^2} - na_{11^2} = \sum_{j=1}^{a} \frac{T_{i.^2}}{nb} - \frac{G^2}{nab}$$

Similarly the effect of B is

$$\sum na_{1j^2} - na_{11^2} = \sum_{j=1}^{b} \frac{T_{.j^2}}{na} - \frac{G^2}{nab}$$

The interaction SS is given by

$$\sum_{i=1}^{a} \sum_{j=1}^{b} \frac{T_{ij^2}}{n} - \sum_{i=1}^{a} \frac{T_{i.^2}}{nb} - \sum_{j=1}^{b} \frac{T_{.j^2}}{na} + \frac{G^2}{nab}$$

This gives the ANOVA table

Source	DF	SS
Mean	1	$\frac{G^2}{nab}$
A	a-1	$\sum_{i=1}^{a}\frac{T_{i.^2}}{nb}-\frac{G^2}{nab}$
B	b-1	$\sum_{j=1}^{a}\frac{T_{.j^2}}{nb}-\frac{G^2}{nab}$
A x B	(a-1)(b-1)	$\sum_{i=1}^{a}\sum_{j=1}^{b}\frac{T_{ij^2}}{n}-\sum_{i=1}^{a}\frac{T_{i.^2}}{nb}-\sum_{j=1}^{b}\frac{T_{.j^2}}{na}+\frac{G^2}{nab}$
Error	(n-1)ab	subtraction
Total	nab	$y'y$

Note that the computer output will resemble the table below, since the total has been corrected for the mean (minus 1 degree of freedom), the mean row has been deleted.

Source	DF	SS
A	a-1	same
B	b-1	as
AB	(a-1)(b-1)	above
Error	(n-1)ab	
Total (corrected)	nab – 1	$y'y-\frac{G^2}{nab}$

16.2 TESTING INDIVIDUAL CONTRASTS (BALANCED CASE)

We wish to test if a particular treatment contrasts for A, α_{i1} exists. We must first test that this contrast is the same over levels of B: Thus we first test the hypothesis

$$H_0 : \alpha_{12} = \alpha_{13} = \ldots = \alpha_{ib} = 0$$

If this contrast interacts we know that the contrast must exist. Breaking things up completely we get the following ANOVA table.

Source	DF	SS
Mean	1	na_{11}^2
Contrast1	1	na_{21}^2
.	.	.
. due to A	.	.
.	.	.
Contrast a-1	1	na_{a1}^2
Contrast 1	1	na_{12}^2
.	.	.
. due to B	.	.
.	.	.
Contrast b-1	1	na_{b}^2
Contrast 1	1	na_{22}^2
.	.	.
.	.	.
.	.	.
Contrast (a-1)(b-1)	1	na_{ab}^2
Residual	(n-1)ab	subt
Total	nab	$y'y$

16.3 Example

Consider the following 2 factor experiment based on Gebotys and Roberts (1988) where a social psychologist is interested in the effect of a type of crime with 3 levels.

A_1 - break and enter, A_2 -sexual assault, A_3 - manslaughter

Age with 4 levels

B_1 - 20 years old
B_2 - 30 years old
B_3 - 40 years old
B_4 - 50 years old

on the sentencing (in months) of offenders. Short descriptions which factorially combined the two factors were given to subjects who were asked to sentence the offender. Each person only responded to one description. There are two people per treatment for a total sample of 3x4x2 = 24.

Results

	B_1	B_2	B_3	B_4
A_1	49 39	50 55	43 38	53 48
A_2	55 41	67 58	53 42	85 73
A_3	66 68	85 92	69 62	85 99

(a) carry out an ANOVA to test for interactions, then effects due to B and A.
(b) carry out an ANOVA to test for differences between B_1 and B_2;
$B_{1,}\, B_2$ and B_3
$B_{1,}\, B_{2,}\, B_3$ and B_4

A_1 and A_2
and
$A_{1,}\; A_2$ and $A_{3.}$

$T_{11} = 88$	$T_{12} = 105$	$T_{13} = 81$	$T_{14} = 101$
$T_{21} = 96$	$T_{22} = 125$	$T_{23} = 95$	$T_{24} = 158$
$T_{31} = 134$	$T_{32} = 177$	$T_{33} = 131$	$T_{34} = 184$
$T_{.1} = 318$	$T_{.2} = 407$	$T_{.3} = 307$	$T_{.4} = 443$
$T_{1.} = 375$	$T_{2.} = 474$	$T_{3.} = 626$	$T_{4.} = 1475$

$$\frac{G^2}{(2\cdot 3\cdot 4)} = 90651.04167$$

$$\frac{\sum T_{.i^2}}{(2\cdot 3)} = 92878.5 \qquad \frac{\sum T_{i.^2}}{(2\cdot 4)} = 94647.125$$

$$\frac{\sum T_{.j^2}}{2} = 97331.5 \qquad \sum y_{i^2} = 97839$$

This gives the following ANOVA table.

ANOVA

Source	DF	SS	MS	F
Mean	1	90651.041	90651.041	
A	2	3996.083	1998.041	47.244
B	3	2227.458	742.486	17.556
AB	6	456.916	76.152	1.800
Error	12	507.500	42.291	
Total	24			

To test for any interaction $F(6, 12)$, $\alpha = .05 = 3.00$ therefore since $1.8 < 3.0$ we have no evidence against the null hypothesis of no interaction. To test for the effect due to B the observed $F(3, 12)$, $\alpha = .05 = 3.49$ since 17.56 is greater than 3.49 we have strong evidence against the null hypothesis of no effect due to B. To test for the effect due to A the observed $F(2, 12)$, $\alpha = .05 = 3.89$, since 47.244 is greater than 3.89 we have strong evidence against the null hypothesis of no effect due to A.

$$C_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{bmatrix} \begin{matrix} \sqrt{3} & C_{11} \\ \sqrt{6} & C_{21} \\ \sqrt{2} & C_{31} \end{matrix} \qquad C_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -3 \\ 1 & 1 & -2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{matrix} 2 & C_{12} \\ \sqrt{12} & C_{22} \\ \sqrt{6} & C_{32} \\ \sqrt{2} & C_{42} \end{matrix}$$

Design

The table below identifies treatment means (β_{ij}) on the basis of subscripts.

	B_1	B_2	B_3	B_4
A_1	β_{11}	β_{12}	β_{13}	β_{14}
A_2	β_{21}	β_{22}	β_{23}	β_{24}
A_3	β_{31}	β_{32}	β_{33}	β_{34}

The design matrix is given by

$C = C_1 \otimes C_2$

			β_{11}	β_{12}	β_{13}	β_{14}	β_{21}	β_{22}	β_{23}	β_{24}	β_{31}	β_{32}	β_{33}	β_{34}	Div
Mean	$C_{11} \otimes C_{12}$	(11)	1	1	1	1	1	1	1	1	1	1	1	1	$\sqrt{12}$
B_1	$C_{11} \otimes C_{22}$	(12)	1	1	1	-3	1	1	1	-3	1	1	1	-3	$\sqrt{36}$
B_2	$C_{11} \otimes C_{32}$	(13)	1	1	-2	0	1	1	-2	0	1	1	-2	0	$\sqrt{18}$
B_3	$C_{11} \otimes C_{42}$	(14)	1	-1	0	0	1	-1	0	0	1	-1	0	0	$\sqrt{6}$
A_1	$C_{21} \otimes C_{12}$	(21)	1	1	1	1	1	1	1	1	-2	-2	-2	-2	$\sqrt{24}$
A_1B_1	$C_{21} \otimes C_{22}$	(22)	1	1	1	-3	1	1	1	-3	-2	-2	-2	6	$\sqrt{72}$
A_1B_2	$C_{21} \otimes C_{32}$	(23)	1	1	-2	0	1	1	-2	0	-2	-2	4	0	$\sqrt{24}$
A_1B_3	$C_{21} \otimes C_{42}$	(24)	1	-1	0	0	1	-1	0	0	-2	-2	0	0	$\sqrt{12}$
A_2	$C_{31} \otimes C_{12}$	(31)	1	1	1	1	-1	-1	-1	-1	0	0	0	0	$\sqrt{8}$
A_2B_1	$C_{31} \otimes C_{22}$	(32)	1	1	1	-3	-1	-1	-1	3	0	0	0	0	$\sqrt{24}$
A_2B_2	$C_{31} \otimes C_{32}$	(33)	1	-2	-2	0	-1	-1	2	0	0	0	0	0	$\sqrt{12}$
A_2B_3	$C_{31} \otimes C_{42}$	(34)	1	0	0	0	-1	1	0	0	0	0	0	0	$\sqrt{4}$

The population contrasts α are

$$\alpha = C\beta$$

$$\alpha_{11} = (\beta_{11} + \beta_{12} + \beta_{13} + \beta_{14} + \beta_{21} + \beta_{22} + \beta_{23} + \beta_{24} + \beta_{31} + \beta_{32} + \beta_{33} + \beta_{34}) / \sqrt{12}$$
$$\alpha_{12} = (\beta_{11} + \beta_{12} + \beta_{13} - 3\beta_{14} + \beta_{21} + \beta_{22} + \beta_{23} - \beta_{24} + \beta_{31} + \beta_{32} + \beta_{33} - \beta_{34}) / \sqrt{36}$$
$$\alpha_{13} = (\beta_{11} + \beta_{12} - 2\beta_{13} + \beta_{21} + \beta_{22} - 2\beta_{23} + \beta_{31} + \beta_{32} - 2\beta_{33}) / \sqrt{18}$$
$$\alpha_{14} = (\beta_{11} - \beta_{12} + \beta_{21} - \beta_{22} + \beta_{31} - \beta_{32}) / \sqrt{6}$$
$$\alpha_{21} = (\beta_{11} + \beta_{12} + \beta_{13} + \beta_{14} + \beta_{21} + \beta_{22} + \beta_{23} + \beta_{24} - 2\beta_{31} - 2\beta_{32} - 2\beta_{33} - 2\beta_{34}) / \sqrt{24}$$
$$\alpha_{22} = (\beta_{11} + \beta_{12} + \beta_{13} - 3\beta_{14} + \beta_{21} + \beta_{22} + \beta_{23} - 3\beta_{24} - 2\beta_{31} - 2\beta_{32} - 2\beta_{33} + 6\beta_{34}) / \sqrt{72}$$
$$\alpha_{23} = (\beta_{11} + \beta_{12} - 2\beta_{13} + \beta_{21} + \beta_{22} - 2\beta_{23} - 2\beta_{31} - 2\beta_{32} + 4\beta_{33}) / \sqrt{24}$$
$$\alpha_{24} = (\beta_{11} - \beta_{12} + \beta_{21} - \beta_{22} - 2\beta_{31} - 2\beta_{33}) / \sqrt{12}$$
$$\alpha_{31} = (\beta_{11} + \beta_{12} + \beta_{13} + \beta_{14} - \beta_{21} - \beta_{22} - \beta_{23} - \beta_{24}) / \sqrt{8}$$
$$\alpha_{32} = (\beta_{11} + \beta_{12} + \beta_{13} - 3\beta_{14} - \beta_{21} - \beta_{22} - \beta_{23} + 3\beta_{24}) / \sqrt{24}$$
$$\alpha_{33} = (\beta_{11} + \beta_{12} - 2\beta_{13} - \beta_{21} - \beta_{22} + 2\beta_{23}) / \sqrt{12}$$
$$\alpha_{34} = (\beta_{11} - \beta_{12} - \beta_{21} + \beta_{22}) / \sqrt{4}$$

Least Squares estimates of α are:

$$\alpha_{11} = (T_{11} + T_{12} + T_{13} + T_{14} + T_{21} + T_{22} + T_{23} + T_{24} + T_{31} + T_{32} + T_{33} + T_{34}) / 2\bullet\sqrt{12}$$
$$\alpha_{12} = (T_{11} + T_{12} + T_{13} - 3T_{14} + T_{21} + T_{22} + T_{23} - T_{24} + T_{31} + T_{32} + T_{33} - T_{34}) / 2\bullet\sqrt{36}$$
$$\alpha_{13} = (T_{11} + T_{12} - 2T_{13} + T_{21} + T_{22} - 2T_{23} + T_{31} + T_{32} - 2T_{33}) / 2\bullet\sqrt{18}$$
$$\alpha_{14} = (T_{11} - T_{12} + T_{21} - T_{22} + T_{31} - T_{32}) / 2\bullet\sqrt{6}$$
$$\alpha_{21} = (T_{11} + T_{12} + T_{13} + T_{14} + T_{21} + T_{22} + T_{23} + T_{24} - 2T_{31} - 2T_{32} - 2T_{33} - 2T_{34}) / 2\bullet\sqrt{24}$$
$$\alpha_{22} = (T_{11} + T_{12} + T_{13} - 3T_{14} + T_{21} + T_{22} + T_{23} - 3T_{24} - 2T_{31} - 2T_{32} - 2T_{33} + 6T_{34}) / 2\bullet\sqrt{72}$$
$$\alpha_{23} = (T_{11} + T_{12} - 2T_{13} + T_{21} + T_{22} - 2T_{23} - 2T_{31} - 2T_{32} + 4T_{33}) / 2\bullet\sqrt{24}$$
$$\alpha_{24} = (T_{11} - T_{12} + T_{21} - T_{22} - 2T_{31} - 2T_{33}) / 2\bullet\sqrt{12}$$
$$\alpha_{31} = (T_{11} + T_{12} + T_{13} + T_{14} - T_{21} - T_{22} - T_{23} - T_{24}) / 2\bullet\sqrt{8}$$
$$\alpha_{32} = (T_{11} + T_{12} + T_{13} - 3T_{14} - T_{21} - T_{22} - T_{23} + 3T_{24}) / 2\bullet\sqrt{24}$$
$$\alpha_{33} = (T_{11} + T_{12} - 2T_{13} - T_{21} - T_{22} + 2T_{23}) / 2\bullet\sqrt{12}$$
$$\alpha_{34} = (T_{11} - T_{12} - T_{21} + T_{22}) / 2\bullet\sqrt{4}$$

Substituting Totals above, we obtain the values below.

$a_{11} = 212.84$	$2a_{11}^2 = 90651.04167$
$a_{12} = -24$	$2a_{12}^2 = 1225.125$
$a_{13} = 13.081$	$2a_{13}^2 = 342.25$
$a_{14} = -18.167$	$2a_{14}^2 = 660.08333$
$a_{21} = -41.131$	$2a_{21}^2 = 3383.520833$
$a_{22} = 1.944$	$2a_{22}^2 = 7.5625$
$a_{23} = -3$	$2a_{23}^2 = 18$
$a_{24} = 5.77$	$2a_{24}^2 = 66.66666$
$a_{31} = -17.5$	$2a_{31}^2 = 612.5625$
$a_{32} = 13.166$	$2a_{32}^2 = 346.6875$
$a_{33} = 0$	$2a_{33}^2 = 0$
$a_{34} = 3$	$2a_{34}^2 = 18$

$$\sum 2a_{ij^2} = 97331.5 = \sum \frac{T_{ij^2}}{2}$$ and this shows we have done the arithmetic correctly.

We obtain the following ANOVA table.

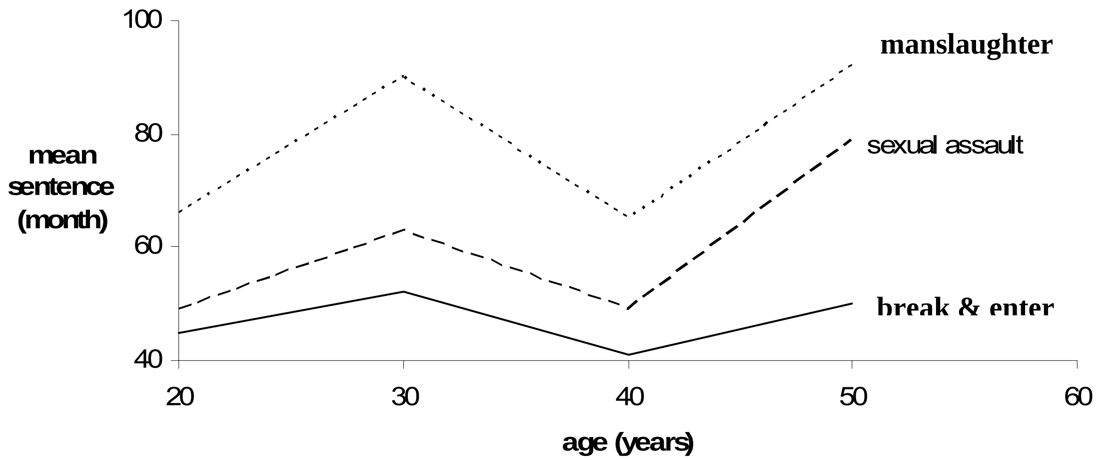
ANOVA

Source	Interpretation
α_{11}	Mean
α_{12}	Contrast 1 for B
α_{13}	Contrast 2 for B
α_{14}	Contrast 3 for B
α_{21}	Contrast 1 for A
α_{31}	Contrast 2 for A
α_{22}	Contrast 1 for A by contrast 1 for B
α_{23}	Contrast 1 for A by contrast 2 for B
α_{24}	Contrast 1 for A by contrast 3 for B
α_{32}	Contrast 2 for A by contrast 1 for B
α_{33}	Contrast 2 for A by contrast 2 for B
α_{34}	Contrast 2 for A by contrast 3 for B

ANOVA

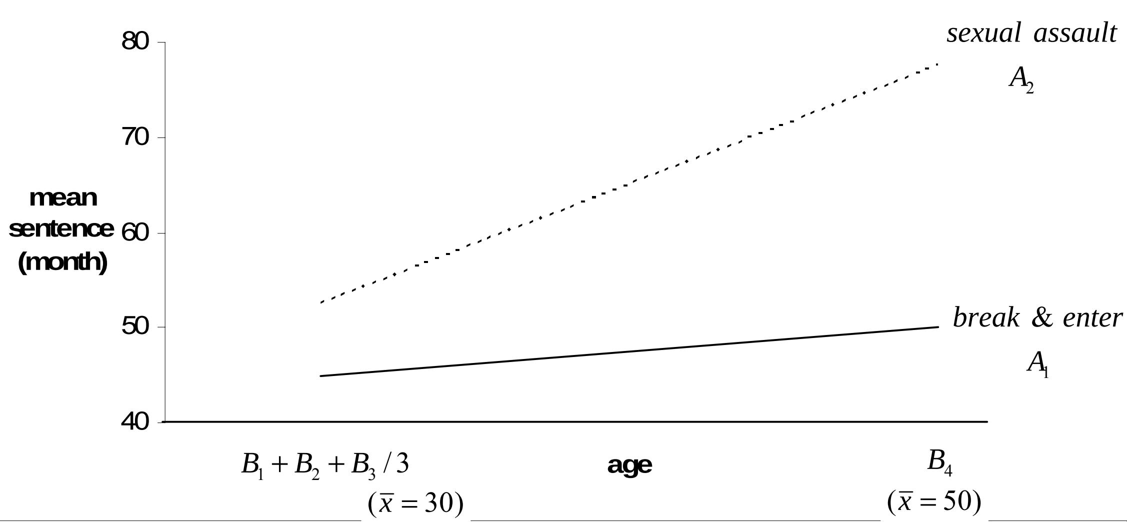
Source	DF	SS	MS	F
α_{11} = Mean	1	90651.04167	90651.04167	
$\alpha_{12} = B_1$	1	1225.125	1225.125	28.97
$\alpha_{13} = B_2$	1	342.25	342.25	8.09
$\alpha_{14} = B_3$	1	660.083333		15.63
$\alpha_{21} = A_1$	1	3383.520833		80.00
$\alpha_{31} = A_2$	1	7.5625		14.48
$\alpha_{22} = A_1B_1$	1	18		.18
$\alpha_{23} = A_1B_2$	1	66.66666		.43
$\alpha_{24} = A_1B_3$	1	612.5625		1.58
$\alpha_{32} = A_2B_1$	1	346.6875		8.20
$\alpha_{33} = A_2B_2$	1	0		0
$\alpha_{34} = A_2B_3$	1	18 42.29166		.43
Error	12	42.29166		
Total	24			

A graph of the means is given below.



The tests of significance are all performed using the
F(1, 12) *dist*ⁿ and the critical values are
$F_{.10}(1, 12) = 3.18$
$F_{.05}(1, 12) = 4.75$
$F_{.005}(1, 12) = 11.8$

We test the interactions first. The A_2B_1 interaction is significant. We have evidence against the null hypothesis that contrast 2 of A is not the same when B is at level B_4 as it is at levels B_1, B_2, B_3 at α=.05 say. (i.e. $F_{obs} = 8.20 > 4.75$)
This implies immediately that contrast 2 of A and contrast 1 of B are important. Further we have the importance of contrast 1 of A and contrast 2 and 1 of B at $\alpha = .05$. A graph of the means is given below.



A 95% CI for σ^2 is given by

$$\left[\frac{12s^2}{X^2{}_{.025}\,12}, \frac{12s^2}{X^2{}_{.975}\,12}\right]$$

[21.746, 115.241]

where $s^2 = 42.29166$

12 is the degrees of freedom for error and X are the critical values for the $\chi^2{}_{12}$ distribution.

C.I. for the contrast α_{ij} is obtained from

$$\left[a_{ij} \pm \frac{s}{\sqrt{2}t_{\frac{1-\alpha}{2}}(12)}\right]$$

where the error degrees of freedom is 12 and the number of observations per treatment is 2.

16.4 SPSS COMMANDS

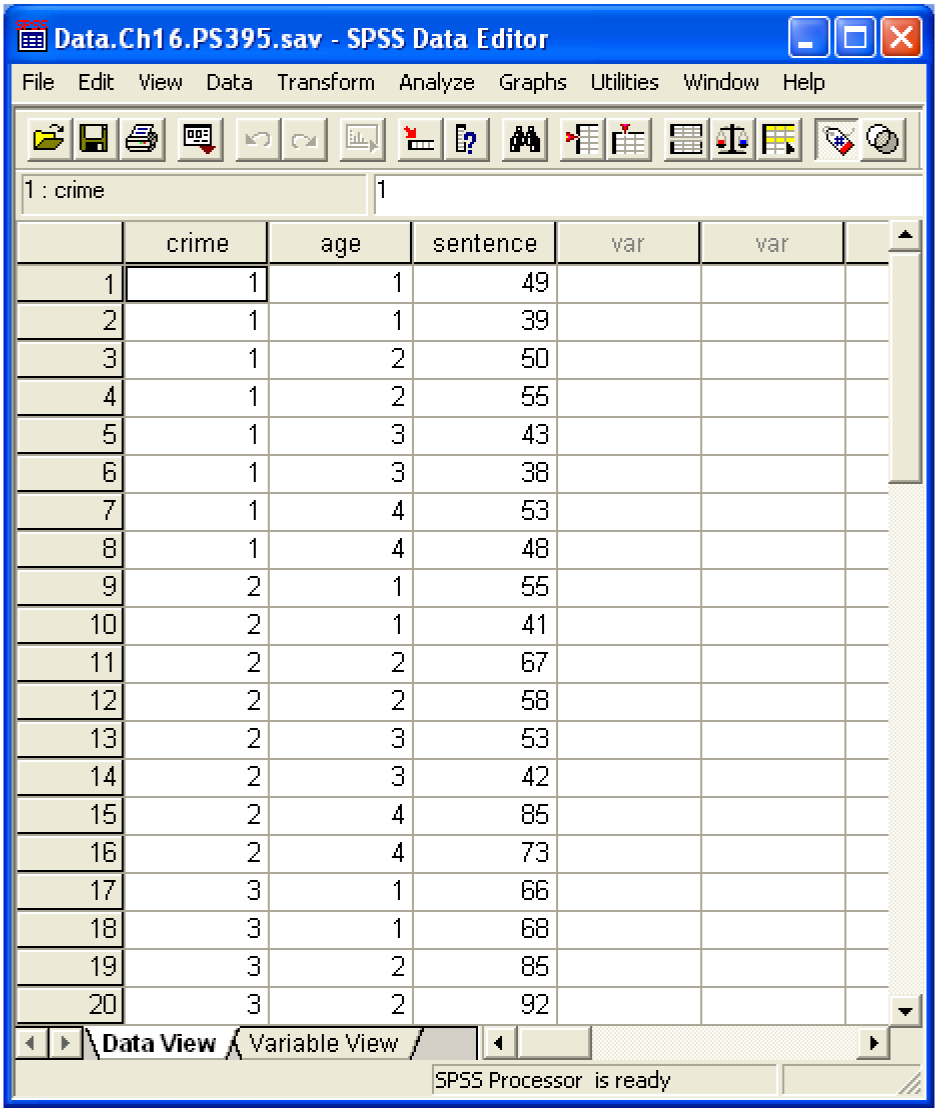
The following SPSS program implements the above analysis using the MANOVA procedure. Please refer back to your chapter 14 notes for the details on how to access a Syntax file.

If you prefer to enter you data into the Syntax window instead of the Data Editor, enter the following commands and data into the Syntax Window:

```
data list/ crime 1 age 3 sent 5-6
begin data
1 1 49
1 1 39
1 2 50
1 2 55
1 3 43
1 3 38
1 4 53
1 4 48
2 1 55
2 1 41
2 2 67
2 2 58
2 3 53
2 3 42
2 4 85
2 4 73
3 1 66
3 1 68
3 2 85
```

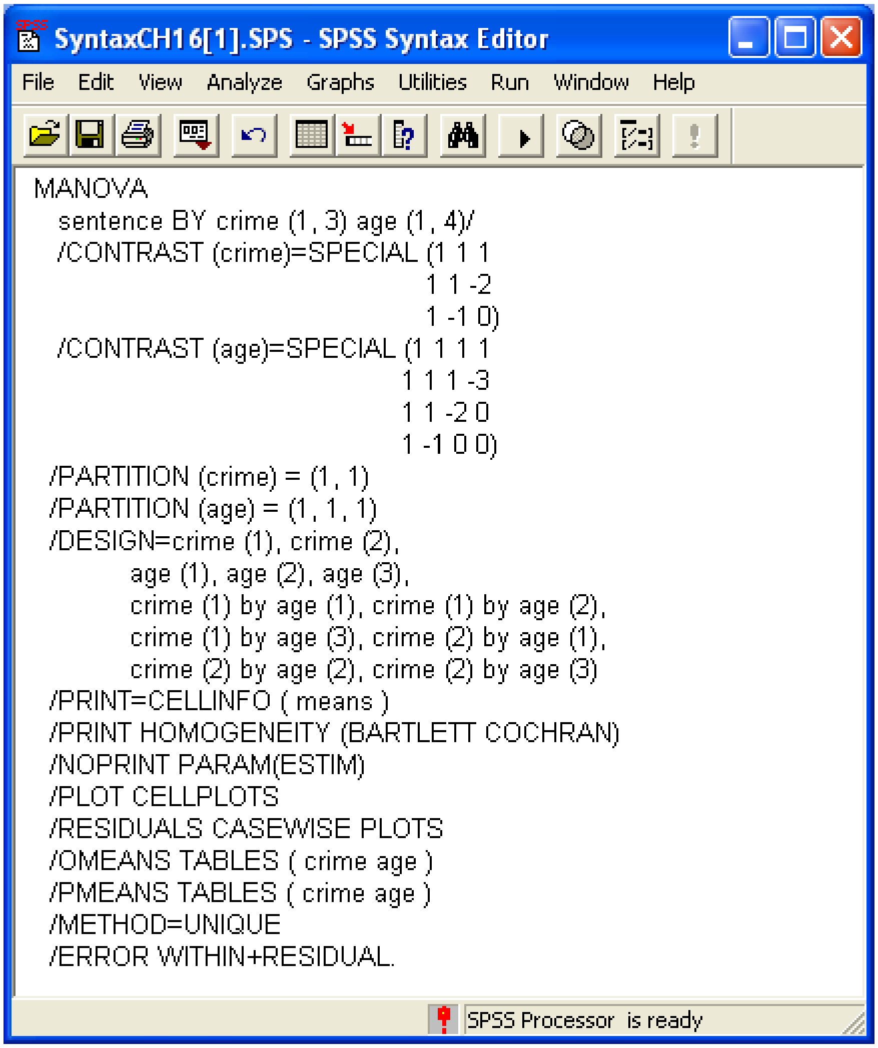
```
3 2 92
3 3 69
3 3 62
3 4 85
3 4 99
end data
MANOVA
   sentence BY crime (1, 3) age (1, 4)/
   /CONTRAST (crime)=SPECIAL (1 1 1
                              1 1 -2
                              1 -1 0)
   /CONTRAST (age)=SPECIAL (1 1 1 1
                            1 1 1 -3
                            1 1 -2 0
                            1 -1 0 0)
  /PARTITION (crime) = (1, 1)
  /PARTITION (age) = (1, 1, 1)
  /DESIGN=crime (1), crime (2),
          age (1), age (2), age (3),
          crime (1) by age (1), crime (1) by age (2),
          crime (1) by age (3), crime (2) by age (1),
          crime (2) by age (2), crime (2) by age (3)
  /PRINT=CELLINFO ( means )
  /PRINT HOMOGENEITY (BARTLETT COCHRAN)
  /NOPRINT PARAM(ESTIM)
  /PLOT CELLPLOTS
  /RESIDUALS CASEWISE PLOTS
  /OMEANS TABLES ( crime age )
  /PMEANS TABLES ( crime age )
  /METHOD=UNIQUE
           /ERROR WITHIN+RESIDUAL.
```

If you prefer to enter your data directly into the Data Editor, here is what your data file should look like:



	crime	age	sentence	var	var
1	1	1	49		
2	1	1	39		
3	1	2	50		
4	1	2	55		
5	1	3	43		
6	1	3	38		
7	1	4	53		
8	1	4	48		
9	2	1	55		
10	2	1	41		
11	2	2	67		
12	2	2	58		
13	2	3	53		
14	2	3	42		
15	2	4	85		
16	2	4	73		
17	3	1	66		
18	3	1	68		
19	3	2	85		
20	3	2	92		

After you have entered your data into the Data Editor, you may begin entering your commands into the Syntax windw. Your Syntax window should appear as follows:


SyntaxCH16[1].SPS - SPSS Syntax Editor

File Edit View Analyze Graphs Utilities Run Window Help

```
MANOVA
  sentence BY crime (1, 3) age (1, 4)/
  /CONTRAST (crime)=SPECIAL (1 1 1
                             1 1 -2
                             1 -1 0)
  /CONTRAST (age)=SPECIAL (1 1 1 1
                           1 1 1 -3
                           1 1 -2 0
                           1 -1 0 0)
 /PARTITION (crime) = (1, 1)
 /PARTITION (age) = (1, 1, 1)
 /DESIGN=crime (1), crime (2),
        age (1), age (2), age (3),
        crime (1) by age (1), crime (1) by age (2),
        crime (1) by age (3), crime (2) by age (1),
        crime (2) by age (2), crime (2) by age (3)
 /PRINT=CELLINFO ( means )
 /PRINT HOMOGENEITY (BARTLETT COCHRAN)
 /NOPRINT PARAM(ESTIM)
 /PLOT CELLPLOTS
 /RESIDUALS CASEWISE PLOTS
 /OMEANS TABLES ( crime age )
 /PMEANS TABLES ( crime age )
 /METHOD=UNIQUE
 /ERROR WITHIN+RESIDUAL.
```

SPSS Processor is ready

Means and standard deviations are printed below. These are useful for plotting the graphs of significant effects.

Cell Means and Standard Deviations
Variable .. SENTENCE

FACTOR	CODE	Mean	Std. Dev.	N
CRIME	1			
AGE	1	44.000	7.071	2
AGE	2	52.500	3.536	2
AGE	3	40.500	3.536	2
AGE	4	50.500	3.536	2
CRIME	2			
AGE	1	48.000	9.899	2
AGE	2	62.500	6.364	2
AGE	3	47.500	7.778	2
AGE	4	79.000	8.485	2
CRIME	3			
AGE	1	67.000	1.414	2
AGE	2	88.500	4.950	2
AGE	3	65.500	4.950	2
AGE	4	92.000	9.899	2
For entire sample		61.458	17.678	24

The fine ANOVA table is printed below. The contrasts and interactions are printed beginning with the error or residual term(WITHIN CELLS). Note that of the interactions, the CRIME(2) by AGE(1) contrast is significant, $F(1,12) = 8.20$, $p = .014$. The means of this interaction would be graphed.

Tests of Significance for SENTENCE using UNIQUE sums of squares

Source of Variation	SS	DF	MS	F	Sig of F
WITHIN+RESIDUAL	507.50	12	42.29		
CRIME(1)	3383.52	1	3383.52	80.00	.000
CRIME(2)	612.56	1	612.56	14.48	.003
AGE(1)	1225.13	1	1225.13	28.97	.000
AGE(2)	342.25	1	342.25	8.09	.015
AGE(3)	660.08	1	660.08	15.61	.002
CRIME(1) BY AGE(1)	7.56	1	7.56	.18	.680
CRIME(1) BY AGE(2)	18.00	1	18.00	.43	.526
CRIME(1) BY AGE(3)	66.67	1	66.67	1.58	.233
CRIME(2) BY AGE(1)	346.69	1	346.69	8.20	.014
CRIME(2) BY AGE(2)	.00	1	.00	.00	1.000
CRIME(2) BY AGE(3)	18.00	1	18.00	.43	.526
(Model)	6680.46	11	607.31	14.36	.000
(Total)	7187.96	23	312.52		

R-Squared = .929
Adjusted R-Squared = .865

The y, $\hat{y}$, e_i and standardized residuals are printed below for the model. The assumptions of normality, homogeneous variance etc. will be examined using the residual plots.

Observed and Predicted Values for Each Case
Dependent Variable.. SENTENCE

Case No.	Observed	Predicted	Raw Resid.	Std Resid.
1	49.000	44.000	5.000	.769
2	39.000	44.000	-5.000	-.769
3	50.000	52.500	-2.500	-.384
4	55.000	52.500	2.500	.384
5	43.000	40.500	2.500	.384
6	38.000	40.500	-2.500	-.384
7	53.000	50.500	2.500	.384
8	48.000	50.500	-2.500	-.384
9	55.000	48.000	7.000	1.076
10	41.000	48.000	-7.000	-1.076
11	67.000	62.500	4.500	.692
12	58.000	62.500	-4.500	-.692
13	53.000	47.500	5.500	.846
14	42.000	47.500	-5.500	-.846
15	85.000	79.000	6.000	.923
16	73.000	79.000	-6.000	-.923
17	66.000	67.000	-1.000	-.154
18	68.000	67.000	1.000	.154
19	85.000	88.500	-3.500	-.538
20	92.000	88.500	3.500	.538
21	69.000	65.500	3.500	.538
22	62.000	65.500	-3.500	-.538
23	85.000	92.000	-7.000	-1.076
24	99.000	92.000	7.000	1.076

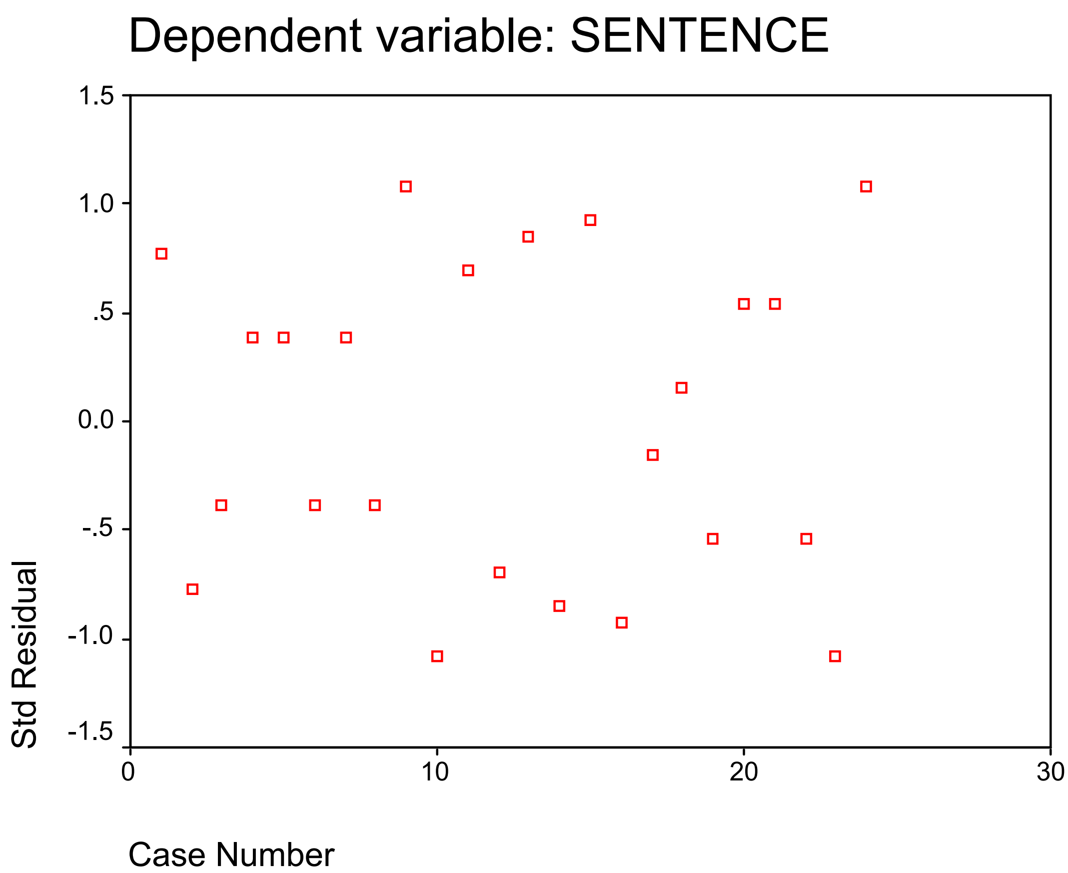
The coarse ANOVA table is listed below. Note that the CRIME by AGE interaction is not significant, $F(6,12) = 1.80$, $p = .18$. The fine table analysis revealed that one contrast was significant of the 6 possible orthogonal possibilities.

Tests of Significance for SENTENCE using UNIQUE sums of squares

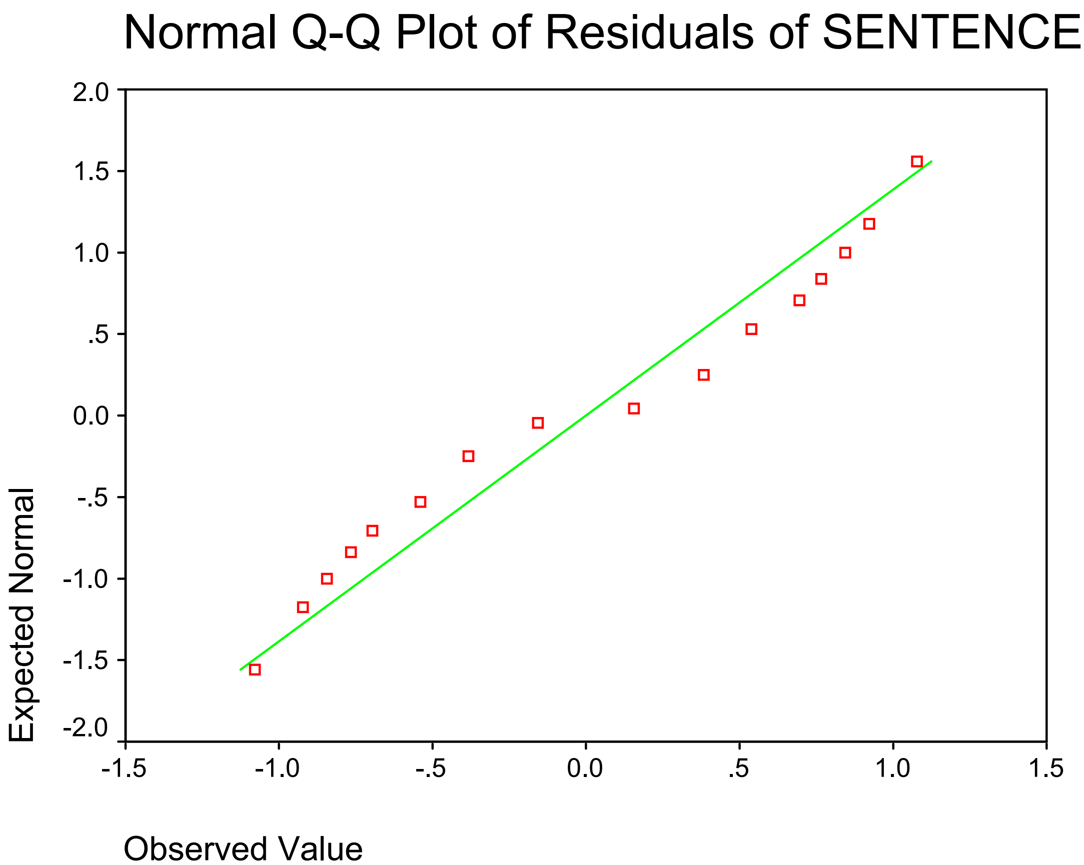
Source of Variation	SS	DF	MS	F	Sig of F
WITHIN+RESIDUAL	507.50	12	42.29		
CRIME	3996.08	2	1998.04	47.24	.000
AGE	2227.46	3	742.49	17.56	.000
CRIME BY AGE	456.92	6	76.15	1.80	.182
(Model)	6680.46	11	607.31	14.36	.000
(Total)	7187.96	23	312.52		

R-Squared = .929
Adjusted R-Squared = .865

The plot of case number vs. standardized e_i clearly indicates a band pattern with no unusual features.



The normal probability plot below approximates a line indicating that the data is normal.



As you can see the direct product is very useful in the data analysis phase of experimental design. Remember if different contrast matricies are selected then the overall design matrix will be different and consequently the analysis will be different. The researcher preplans the design and it is her decision as to what treatments are selected and compared. The comparison of treatments is through the contrast matricies of the factors and direct product of thes matricies which result in an overall design matrix for the experiment.

Click here for a SPSS windows version of a two way anova.

16.5 Exercises

1. The weight gains for individuals in a clinical study were recorded under the following 6 treatments, one person per treatment for N = 60. The two factors were:

   Protein (3 levels): Beef, Cereal, Pork
   Amount (2 levels): High, Low

The data is given below.

Gains in Weight Under Six Diets

High Protein			Low Protein		
Beef	Cereal	Pork	Beef	Cereal	Pork
73	98	94	90	107	49
102	74	79	76	95	82
118	56	96	90	97	73
104	111	98	64	80	86
81	95	102	86	98	81
107	88	102	51	74	97
100	82	108	72	74	106
87	77	91	90	67	70
117	86	120	95	89	61
111	92	105	78	58	82

   a. Perform the appropriate ANOVA. State your conclusions clearly.

   b. Contrast Animal vs Vegetable protein, and Pork vs Beef. Are they orthogonal? Repeat the complete ANOVA table including contrasts. State your conclusions clearly.

   c. Are the residuals reasonable?

2. See problem 4, 14.8 and perform a two-way analysis of variance on the data.

   a. Clearly state your conclusions.

   b. Comment on the residuals.