# 23. REPEATED MEASURES TYPES OF DESIGNS

## 23.1 INTRODUCTION

We treat repeated measures types of designs as a unique topic since in pscyhology, social work, education...etc. these types of designs are extremely popular. These designs can be viewed as having two components. They can be considered as examples of nested designs with specific assumptions on the correlations between observations. We will briefly discuss both points of view. Psychologists, when conducting studies that use humans and animals, commonly take observations on the same person/animal a number of times, say j = 1, 2, psychology when studying how learning occurs over time. Note that in contrast to the randomized block design, treatments are not randomly allocated to blocks.

| | Time | | | |
|---|---|---|---|---|
| Subject | 1 | 2 | . . . | t |
| $S_1$ | $y_{11}$ | $y_{12}$ | . . . | $y_{1t}$ |
| $S_2$ | $t_{21}$ | $y_{22}$ | . . . | $y_{2t}$ |
| . | . | | | . |
| . | . | | | . |
| . | . | | | . |
| $S_n$ | $y_{n1}$ | $y_{n2}$ | . . . | $y_{nt}$ |

## 23.2 NESTED OR UNIVARIATE APPROACH

The above design is very similar to the two factor factorial model with one observation per treatment. The effect of time is seen within people (subjects).

The EMS table would look like the following.

| | | n | t | 1 | |
|---|---|---|---|---|---|
| | | R | F | R | |
| Source | DF | i | j | k | EMS |
| $S_i$ | n-1 | 1 | t | 1 | $\sigma_e^2 + t\sigma_S^2$ |
| $T_j$ | t-1 | n | 0 | 1 | $\sigma_e^2 + \sigma_{ST}^2 + n\sigma_T^2$ |
| $ST_{ij}$ | (n-1)(t-1) | 1 | 0 | 1 | $\sigma_e^2 + \sigma_{ST}^2$ |
| $E_{k[ij]}$ | 0 | 1 | 1 | 1 | $\sigma_e^2$ (not available) |

Since there is only one observation per cell, the error term, e, is not estimable. Treating the ST interaction term as error is the proper F test for the Time (T) factor.

The ANOVA table is usually reported as follows.

| Source | DF | MS | F |
|---|---|---|---|
| Between Subjects $(S_i)$ | n-1 | | |
| Within Subjects | | | |
| $T_j$ | t-1 | MST | $\frac{MST}{MSE}$ |
| Error | (n-1)(t-1) | MSE | |
| Total | nt-1 | | |

The proper F test of the Time factor is $\frac{MST}{MSE}$.

## 23.3 CORRELATION ASSUMPTIONS

As mentioned previously given that *between* person variablility is greater than *within* person variability, it is more efficient for the psychologist to block on people.

The ANOVA model for this design includes an effect due to Time and Subjects as mentioned previously, however, the assumption of is stressed. In other words the correlation of $y_{ij}$ is constant and takes the form

$$\begin{vmatrix} 1 & \rho & . & . & . & . & \rho \\ \rho & 1 & & & & & . \\ . & & . & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ \rho & . & . & . & . & . & 1 \end{vmatrix}$$

The symmetry condition can be assessed using the Huynh and Feldt (1970) procedure. For example, if observations taken closely in time were more highly correlated than those taken further apart, the compound symmetry condition would be violated. SPSS gives a test of significance called the Mauchly test of spericity and corresponding epsilon multipliers that correct for violations in this assumption. Note the inclusion of the *compound symmetry* assumption in addition to the other linear model assumptions,ie normality,etc. Sometimes a different **MULTIUNIVARIATE APPROACH** is taken to this same problem. Consider the t times as t variables recorded on each person. With this approach one assumes the variables are distributed according to a multivariate normal distribution and there is no assumption of symmetry or circularity. Unfortunately for the student both the univariate and multivariate material are printed in spss and they are mixed in the printout.

**23.4 EXAMPLE**

Siegel, P.S., Activity level as a function of physically forced inaction, *J. of Psychol*, 1946, 285-291 examined how activity level in rats was a function of confinement. An animal was confined for 0, 6, 12 and 24 hours and activity recorded after the period of confinement. The data is given below.

| Rat | 0 Hours | 6 Hours | 12 Hours | 24 Hours |
|---|---|---|---|---|
| 1 | 232 | 244 | 213 | 272 |
| 2 | 216 | 212 | 191 | 269 |
| 3 | 112 | 62 | 69 | 119 |
| 4 | 219 | 119 | 200 | 251 |
| 5 | 292 | 165 | 187 | 287 |
| 6 | 179 | 106 | 189 | 217 |
| 7 | 264 | 271 | 354 | 365 |
| 8 | 247 | 260 | 295 | 305 |
| 9 | 259 | 241 | 196 | 211 |
| 10 | 195 | 118 | 150 | 184 |
| 11 | 140 | 121 | 102 | 136 |
| 12 | 244 | 189 | 229 | 240 |
| 13 | 364 | 326 | 329 | 303 |
| 14 | 302 | 282 | 292 | 362 |
| 15 | 312 | 233 | 225 | 306 |
| 16 | 350 | 283 | 312 | 430 |
| Mean | 245 | 202 | 221 | 266 |

Click here for the SPSS windows analysis of this problem

**Syntax Computer Implementation**

```
data list        /rat 1-3, time 4-5, y 6-9
begin data
01 1 232
01 2 244
01 3 213
01 4 272
02 1 216
02 2 212
02 3 191
02 4 269
03 1 112
03 2 062
03 3 069
03 4 119
04 1 219
04 2 119
04 3 200
```

```
04 4 251
05 1 292
05 2 165
05 3 187
05 4 287
06 1 179
06 2 106
06 3 189
06 4 217
07 1 264
07 2 271
07 3 354
07 4 365
08 1 247
08 2 260
08 3 295
08 4 305
09 1 259
09 2 241
09 3 196
09 4 211
10 1 195
10 2 118
10 3 150
10 4 184
11 1 140
11 2 121
11 3 102
11 4 136
12 1 244
12 2 189
12 3 229
12 4 240
13 1 364
13 2 326
13 3 329
13 4 303
14 1 302
14 2 282
14 3 292
14 4 362
15 1 312
15 2 233
15 3 225
15 4 306
16 1 350
16 2 283
16 3 312
16 4 430
end data
frequency general=all/
```

```
statistics all
        manova y by rat(1,16) time(1,4)/
        print=cellinfo(means)/
        omeans=TABLES(time)/
        residuals=casewise plot/
        design=rat, time/
finish
```

Tables of means are given below.

```
Cell Means and Standard Deviations
 Variable .. Y
      FACTOR              CODE                 Mean  Std. Dev.          N

  RAT                       1
   TIME                      1              232.000       .000          1
   TIME                      2              244.000       .000          1
   TIME                      3              213.000       .000          1
   TIME                      4              272.000       .000          1
  RAT                       2
   TIME                      1              216.000       .000          1
   TIME                      2              212.000       .000          1
   TIME                      3              191.000       .000          1
   TIME                      4              269.000       .000          1
  RAT                       3
   TIME                      1              112.000       .000          1
   TIME                      2               62.000       .000          1
   TIME                      3               69.000       .000          1
   TIME                      4              119.000       .000          1
  RAT                       4
   TIME                      1              219.000       .000          1
   TIME                      2              119.000       .000          1
   TIME                      3              200.000       .000          1
   TIME                      4              251.000       .000          1
  RAT                       5
   TIME                      1              292.000       .000          1
   TIME                      2              165.000       .000          1
   TIME                      3              187.000       .000          1
   TIME                      4              287.000       .000          1
  RAT                       6
   TIME                      1              179.000       .000          1
   TIME                      2              106.000       .000          1
   TIME                      3              189.000       .000          1
   TIME                      4              217.000       .000          1
  RAT                       7
   TIME                      1              264.000       .000          1
   TIME                      2              271.000       .000          1
   TIME                      3              354.000       .000          1
   TIME                      4              365.000       .000          1
  RAT                       8
   TIME                      1              247.000       .000          1
   TIME                      2              260.000       .000          1
   TIME                      3              295.000       .000          1
   TIME                      4              305.000       .000          1
  RAT                       9
   TIME                      1              259.000       .000          1
   TIME                      2              241.000       .000          1
```

```
  TIME                   3              196.000       .000          1
  TIME                   4              211.000       .000          1
 RAT                    10
  TIME                   1              195.000       .000          1
  TIME                   2              118.000       .000          1
  TIME                   3              150.000       .000          1
  TIME                   4              184.000       .000          1
 RAT                    11
  TIME                   1              140.000       .000          1
  TIME                   2              121.000       .000          1
  TIME                   3              102.000       .000          1
  TIME                   4              136.000       .000          1
 RAT                    12
  TIME                   1              244.000       .000          1
  TIME                   2              189.000       .000          1
  TIME                   3              229.000       .000          1
  TIME                   4              240.000       .000          1
 RAT                    13
  TIME                   1              364.000       .000          1
  TIME                   2              326.000       .000          1
  TIME                   3              329.000       .000          1
  TIME                   4              303.000       .000          1
 RAT                    14
  TIME                   1              302.000       .000          1
  TIME                   2              282.000       .000          1
  TIME                   3              292.000       .000          1
  TIME                   4              362.000       .000          1
 RAT                    15
  TIME                   1              312.000       .000          1
  TIME                   2              233.000       .000          1
  TIME                   3              225.000       .000          1
  TIME                   4              306.000       .000          1
 RAT                    16
  TIME                   1              350.000       .000          1
  TIME                   2              283.000       .000          1
  TIME                   3              312.000       .000          1
  TIME                   4              430.000       .000          1
For entire sample                       233.578     79.633         64


Combined Observed Means for TIME
Variable .. Y
         TIME
            1       WGT.   245.43750
                  UNWGT.   245.43750
            2       WGT.   202.00000
                  UNWGT.   202.00000
            3       WGT.   220.81250
                  UNWGT.   220.81250
            4       WGT.   266.06250
                  UNWGT.   266.06250
```

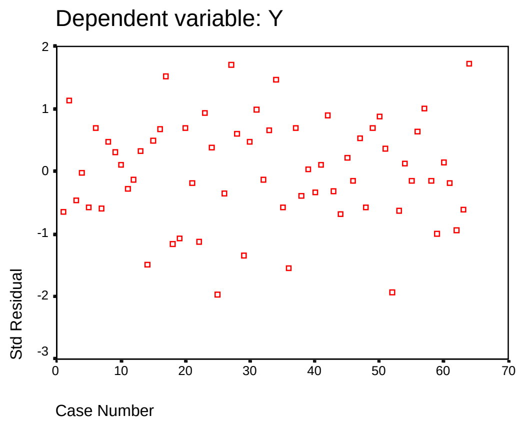
The coarse ANOVA table is given below. The TIME factor is significant with $F(3,45) = 13.01$, $p<.001$. Planned contrasts or multiple comparisons could be used to further examine the means.

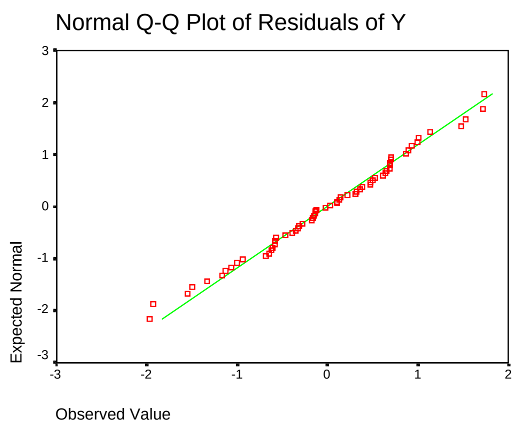
```
Tests of Significance for Y using UNIQUE sums of squares
 Source of Variation          SS      DF        MS        F  Sig of F

 RESIDUAL               43463.45      45    965.85
 RAT                   318347.86      15  21223.19    21.97      .000
 TIME                   37696.30       3  12565.43    13.01      .000

 (Model)               356044.16      18  19780.23    20.48      .000
 (Total)               399507.61      63   6341.39
```

The residuals look reasonable in both plots.




## Another Computer Implementation

Another type of implementation is given below that may prove helpful, note that the data is entered differently.

```
data list/ rat 1-2 h0 4-6 h6 8-10 h12 14-16 h24 18-20
begin data
01 232 244 213 272
02 216 212 291 269
 .  .  .  .  .
 .  .  .  .  .
```

```
 . . . . .
16 350 202 221 266
end data
manova h0 h6 h12 h24
        wsfactors=time(4)/
        wsdesign=time/
        print=cellinfo(means)/
        transform/
        signif(univ,averf)/
        analysis(repeated)/
        design/
finish
```

## 23.5 MORE COMPLEX MODELS

Suppose an investigator wished to compare both a *between* subject and a *within* subject factor. This type of mixed model is very popular in psychology and social work. The within subject factor is usually a repeated measures time factor (T) which could be the pre, post and post-post test scores of an individual. The between subject factor is usually a factor which represents the type of experimental condition that each person undergoes, call this the Groups factor (G). Subjects are nested within groups ($S_{j\,(i)}$ ).

The EMS table would look like

| Source | DF | G<br>F<br>i | n<br>R<br>j | t<br>F<br>k | m<br>R<br>l | EMS |
|---|---|---|---|---|---|---|
| $G_i$ | (g-1) | 0 | n | t | 1 | $\sigma_e^2 + t\,\sigma_S^2 + tn\theta_G$ |
| $S_j(i)$ | g(n-1) | 1 | 1 | t | 1 | $\sigma_e^2 + t\,\sigma_S^2$ |
| $T_k$ | t-1 | g | n | 0 | 1 | $\sigma_e^2 + \sigma_{TS}^2 + tg\theta_T$ |
| $GT_{ik}$ | (g-1)(t-1) | 0 | n | 0 | 1 | $\sigma_e^2 + \sigma_{TS}^2 + n\theta_{GT}$ |
| $TS_{kj(i)}$ (*Error*W) | (t-1)g(n-1) | 1 | 1 | 0 | 1 | $\sigma_e^2 + \sigma_{TS}^2$ |
| $e_l\,[_{ijk}]$ | 0 | 1 | 1 | 1 | 1 | $\sigma_e^2$ (not available) |

The ANOVA table is given below.

**TABLE 1**

| Source | DF | | MS | F |
|---|---|---|---|---|
| Between Subjects (B) | gn-1 | | | |
| $G_i$ | | g-1 | MSG | $\dfrac{MSG}{MSB}$ |

| | | | |
|---|---|---|---|
| $S_{j\,(i)}$ (Error B) | g(n-1) | MSB | |
| Within Subjects (W) | | | |
| $T_k$ | t-1 | MST | $\frac{MST}{MSW}$ |
| $GT_{ik}$ | (g-1)(t-1) | MSGT | $\frac{MSGT}{MSW}$ |
| $TS_{kj(i)}$ | (t-1)g(n-1) | MSW | |
| Total | gnt-1 | | |

There are two error terms, one for the between subject factor G, call this error B, and a within subject error term, error W for the within terms T and TG. The EMS table clearly indicates the proper terms for testing. The above method of breaking down the model is thoroughly reported in Winer and is an example of the nested approach of Section 20.2.

Extensions of this approach are straightforward. For example, consider two fixed factors between and one within repeated.

The ANOVA breakdown would read as follows.

**TABLE 2**

| Source | DF |
|---|---|
| Between Subjects | |
| $A_i$ | a-1 |
| $B_j$ | b-1 |
| $AB_{ij}$ | (a-1)(b-1) |
| $S_{k(ij)}$ (Error B) | ab(n-1) |
| Within Subjects | |
| $C_l$ | (c-1) |
| $AC_{il}$ | (a-1)(c-1) |
| $BC_{ji}$ | (b-1)(c-1) |
| $ABC_{ijl}$ | (a-1)(b-1)(c-1) |
| $CS_{lk(ij)}$ (Error W) | (c-1)ab(n-1) |
| Total | abnc-1 |

The pattern of analysis with two error terms is repeated in the above example. The A, B and AB terms are tested with the between subject error term whereas the C, AC, BC and ABC terms are tested with the within error term.

## 23.6 CORRELATION ASSUMPTIONS FOR UNIVARIATE APPROACH

As in the one factor repeated measures design, the T factor of more complex designs assumes the compound symmetry conditions previously discussed. Let us first examine the one factor between and one within design given in Table 1. Refering to Table 1, the F test of the between factor G is not affected. If the Huynh-Feldt conditions do not hold then Greenhouse and Geisser (1959) and Huynh and Feldt (1970) suggest *adjusted* F values be used to determine significance. As we saw previously a quantity called *epsilon* ( ε) is used to adjust the degrees of freedom of the test. For example to test the effect of the T factor of Table 1 we would compute as before

$$F = \frac{MST}{MSW}$$

however the degrees of freedom for this test are

df numerator = $\varepsilon\,(t-1)$

df denominator = $\varepsilon\,(t-1)g\,(n-1)$

Note that values of ε closer to 0 indicate serious departure from the assumption whereas values closer to 1 indicate less serious violations. When ε = 1 we have the original F test. Epsilon ranges between 0 and 1. The MULTIVARIATE approach does not require the circularity/symettry assumption and as we have seen can readily be applied to the mixed model. SPSS once again mixes the relavent output for both approaches. Remember the different approaches , univariate and multivariate only refer to the within subject variable. The between subject variable is reported the same way for both approaches.

Click here for the SPSS windows method of analysis of the example below.

**23.7 Example**

Duneck, E.R., Learning with Secondary Reinforcement Under Tow Different Strengths of the Relevant Drive, 1949, as reported in Lindquist (1953), examined how two groups of animals (n=10 in each group), one hungry and the other satiated learned a maze task over 20 days. The repeated day factor was collapsed into 4 times. $T_1$ (Days 1-5), $T_2$ (6-10), $T_3$ (11-15) and $T_4$ (16-20). The data are given below.

Trial Categories

| | | $T_1$ | $T_2$ | $T_3$ | $T_4$ |
|---|---|---|---|---|---|
| | Animal | Days 1-5 | Days 6-10 | Days 11-15 | Days 16-20 |
| Hungry ($H_1$) | 1 | 3 | 3 | 3 | 3 |
| | 2 | 2 | 2 | 4 | 4 |
| | 3 | 4 | 2 | 5 | 5 |
| | 4 | 3 | 5 | 5 | 5 |
| | 5 | 2 | 5 | 5 | 4 |
| | 6 | 3 | 5 | 4 | 0 |
| | 7 | 1 | 3 | 4 | 1 |
| | 8 | 4 | 5 | 3 | 3 |
| | 9 | 3 | 5 | 4 | 2 |
| | 10 | 0 | 2 | 1 | 0 |
| Satiated ($H_2$) | 11 | 0 | 1 | 2 | 2 |
| | 12 | 3 | 0 | 2 | 2 |
| | 13 | 2 | 3 | 4 | 4 |
| | 14 | 0 | 1 | 2 | 1 |
| | 15 | 1 | 3 | 5 | 4 |
| | 16 | 1 | 2 | 3 | 2 |
| | 17 | 2 | 5 | 1 | 1 |
| | 18 | 2 | 3 | 4 | 2 |
| | 19 | 3 | 1 | 2 | 1 |
| | 20 | 3 | 2 | 2 | 3 |

**Computer Implementation**

```
data list/ HG 1 anim 3-4 T1 6 T2 8 T3 10 T4 12
begin data
1 1 3 3 3 3
1 2 2 2 4 4
  . . . . .
  . . . . .
  . . . . .
2 20 3 2 2 3
end data
manova T1 T2 T3 T4 by HG(1,2)/
        wsfactors=time(4)/
        wsdesign=time/
        print=cellinfo(means)/
        transform/
```

```
homogeneity(boxm)/
signif(averf)/
analysis(repeated)/
design/
finish
```

The means are given below.

```
Cell Means and Standard Deviations
 Variable .. T1
      FACTOR           CODE                  Mean  Std. Dev.           N

  HG                     1                  2.500      1.269          10
  HG                     2                  1.700      1.160          10
 For entire sample                          2.100      1.252          20

 - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
- -
 Variable .. T2
      FACTOR           CODE                  Mean  Std. Dev.           N

  HG                     1                  3.700      1.418          10
  HG                     2                  2.100      1.449          10
 For entire sample                          2.900      1.619          20

 - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
- -
 Variable .. T3
      FACTOR           CODE                  Mean  Std. Dev.           N

  HG                     1                  3.800      1.229          10
  HG                     2                  2.700      1.252          10
 For entire sample                          3.250      1.333          20

 - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
- -
 Variable .. T4
      FACTOR           CODE                  Mean  Std. Dev.           N

  HG                     1                  2.700      1.889          10
  HG                     2                  2.200      1.135          10
 For entire sample                          2.450      1.538          20
```

The test for homogeneity is not significant, p = .85.

```
Multivariate test for Homogeneity of Dispersion matrices
Boxs M = 7.32300
F WITH (10,1549) DF = .55263, P = .853 (Approx.)
Chi-Square with 10 DF = 5.57361, P = .850 (Approx.)
```

The between subjects HG effect is significant, $F(1,18) = 5.71$, $p = .028$.

```
Tests of Between-Subjects Effects.

 Tests of Significance for T1 using UNIQUE sums of squares
 Source of Variation          SS      DF        MS         F  Sig of F

 WITHIN+RESIDUAL           63.05      18      3.50
 HG                        20.00       1     20.00      5.71      .028
```

The epsilon values are given below and are reasonable.

```
Tests involving 'TIME' Within-Subject Effect.


 Mauchly sphericity test, W =      .63496
 Chi-square approx. =           7.59500 with 5 D. F.
 Significance =                    .180

 Greenhouse-Geisser Epsilon =      .79888
 Huynh-Feldt Epsilon =             .98125
 Lower-bound Epsilon =             .33333
```

The within subjects effect of Time is significant, F(3,54) = 3.82, p = .015.

```
Tests involving 'TIME' Within-Subject Effect.

 AVERAGED Tests of Significance for T using UNIQUE sums of squares
 Source of Variation          SS      DF        MS         F  Sig of F

 WITHIN+RESIDUAL           71.95      54      1.33
 TIME                      15.25       3      5.08      3.82      .015
 HG BY TIME                 3.30       3      1.10       .83      .486
```

**Another Computer Implementation**

```
data list/ HG 1 anim 3-4 time 6 Y 8
begin data
1 1 1 3
 . . .
 . . .
 . . .
1 10 4 0
 . . .
 . . .
 . . .
2 10 4 3
end data
manova Y by HG(1,2) anim(1,10) time(1,4)/
        design=HG vs 1, anim WHG=1, time, HG by time/
        omeans=TABLES(HG,time,HG by time)/
        residuals=casewise plot/
finish
```

### 23.8 Exercises

Hockley (1991) conducted an experiment in which pariticipants studied pairs of words and were later tested for recognition memory for single words (item information) and associations between words (associative information). The principal purpose of this experiment was to determine whether recognition memory is the same or different depending on the nature of the recognition test. Two different types of recognition tests were used. In the yes-no test procedure, a single word. Repeated Measures Types of Designs 431 or a pair of words was presented and participants tried to decide if that word or pair or words appeared in the study list. In the forced-choice test, two single words, or two pairs of words were presented and pariticipants tried to decide which of the two words or pairs had been presented in the study list. The results of the experiment are presented below. Is one test easier than the other or are both types of tests equally difficult?

| | TEST | | | |
|---|---|---|---|---|
| | YES/NO | | F-C | |
| | TYPE | | | |
| | ITEM | ASSOC | ITEM | ASSOC |
| High Accuracy | | | | |
| S1 | 1.97 | 2.36 | 1.81 | 2.09 |
| S5 | 2.48 | 3.22 | 2.48 | 3.29 |
| S6 | 1.43 | 1.12 | 1.09 | 1.19 |
| $\bar{x}$ | 1.96 | 2.23 | 1.79 | 2.19 |
| Low Accuracy | | | | |
| S2 | 1.32 | .95 | 1.14 | .95 |
| S3 | .48 | .18 | .54 | .21 |
| S4 | .92 | .70 | .82 | .58 |
| $\bar{x}$ | .76 | .61 | .83 | .58 |