HANDOUT ON VALIDITY

A measure (e.g. a test, a questionnaire or a scale) is useful if it is reliable and valid. A measure is valid if it measures what it purports to measure. Validity can be assessed in several ways depending on the measure and its use.

1. <u>Content Validity</u>

Content validation is employed when it seems likely that test users will want to draw references from observed test scores to performances on a larger domain of tasks similar to items on the test. Typically, it involves asking expert judges to examine test items and judge the extent to which these items sample a specified performance domain. There are two types of content validity: face validity and logical validity. A test has face validity if an examination of the items leads to the conclusion that the items are measuring what they are supposed to be measuring. Logical or sampling validity is based on a careful comparison of the items to the definition of the domain being measured.

2. <u>Criterion Related Validity</u>

Criterion-related validation is a study of the relationship between test scores and a practical performance criterion that is measurable. The criterion is the thing of interest or the outcome we are concerned about. When a test score, X, can be related to a criterion score, Y, criterion-related validity can be determined. The validity coefficient, ρ_{XY} can be based on a predictive or a concurrent study. A predictive-validity coefficient is obtained by giving the test to all relevant people, waiting a reasonable amount of time, collecting criterion scores, and calculating the validity coefficient. When a test is used to predict future behaviour, predictive validity should be established. A concurrent-validity coefficient is a correlation between test and criterion scores when both measurements are obtained at the same time. Concurrent-validity coefficients are appropriate when the test scores are used to estimate a concurrent criterion rather than to predict a future criterion.

3. <u>Construct Validity</u>

Construct validation is appropriate whenever the test user wants to draw inferences from test scores to a behaviour domain which cannot be adequately represented by a single criterion or completely defined by a universe of content. A test's construct validity is the degree to which it measures the behaviour domain or other theoretical constructs or traits that it was designed to measure. More specifically, construct validity can be understood as the extent to which the behaviour domain or the constructs of theoretical interest have been successfully operationalized. For example, a researcher may be interested in determining clients' satisfaction with health care services. Since "satisfaction with health care services" is a construct which cannot be adequately represented by a criterion or defined by a universe of content, the researcher chooses to develop a questionnaire of 20 items in order to tap the construct "satisfaction" and proceeds to collect the data. The question is ow does the researcher know that what he/she is measuring through the questionnaire is actually and purely clients' satisfaction with health care services and not something else nor a mixture with other constructs such as clients' degree of confidence in the medical profession? In this case, a construct validation is appropriate.

Establishing construct validity is an ongoing process that involves the verification of predictions made about the test scores. Procedures for construct validation may include correlations between test scores and designated criterion variables, differentiation between groups, factor analysis, multitrait-multimethod matrix analysis, or analysis of variance components within the framework of generalizability theory. The following pages will contain introductions and explanations of one of the procedures for determining construct validity: the factor analysis.

a. ***Factorial Validity***

Factorial validity is a form of construct validity that is established through a factor analysis. Factor analysis is a term that represents a large number of different mathematical procedures for analyzing the interrelationships among a set of variables and for explaining these interrelationships in terms of a reduced number of variables, called factors. A factor is a hypothetical variable that influences scores on one or more observed variables. For example, let's look at the following hypothetical correlation matrix:

		TEST		
		1	2	3
	1	1.00	.98	.95
Test	2	.98	1.00	.97
	3	.95	.95	1.00

Although there are three test scores being correlated, it is quite obvious that only one dimension/factor is being measured, because of the high correlations among the test scores[1]. Instead of requiring three scores for each person, one score alone could be sufficient.

[1] This is based on the assumption that tests measuring the same trait should correlate highly, converging on the trait.

The preceding example involves an “eyeball” method of factor analysis. This is possible if the correlation matrix is small and simple. But most of the correlation matrices are more complex and the “eyeball” method of factor analysis is either difficult or unreliable (or both). However, the logic underlying the above simple example in terms of determining the number of factors remains the same for complex cases. This logic is helpful in determining the construct validity of a test, with the help of the SPSS factor procedure.

To go back to the example on “satisfaction with health care services” cited earlier, it is not difficult to envisage that if the 20 - item questionnaire is really a valid measure of the construct “satisfaction with health care services”, a factor analysis on the scores of the 20 - item questionnaire should result in one factor that can explain most of the variances in these 20 items. But if the 20 - item questionnaire is instead measuring two different behaviour domains (e.g. “Satisfaction with health care services” and “confidence in the medical profession”), factor analysis on the scores of the 20 - item questionnaire should result in two factors, with items measuring “satisfaction” having high factor loadings[2] on one factor and items measuring “confidence” loading highly on the remaining factor.

To conclude, factorial validity is one form of construct validity. Factorial validity is assessed by the process of factor analyzing the correlations of scores from selected tests (or individual items in a single test) and obtaining a predicted factor-loading pattern.

Determining Factorial Validity Using SPSS Factor Procedure:

Example 1:

The following illustrative example contains six items extracted from a scale designed to measure adolescents’ attitude towards the use of physical aggressive behaviours in their daily life. Each item in the scale refers to a situation where physical aggressive behaviour is or is not used. Adolescents are asked whether they agree or disagree with each and every item on the scale. Adolescents’ responses to the items are converted to scores of either 1 or 0, where 0 represents disapproval of the use of physical aggressive behaviours. Below are the contents of the six items as well as the scores of 14 adolescents on these six items:

Item No.	Content
1.	When there are conflicts, people won’t listen to you unless you get physically aggressive.

[2] The meaning of factor loadings will be discussed in greater detail in a later section. In the mentime, just imagine that a factor loading is a number which is very much like a correlation coefficient in size and meaning. When a factor analysis is conducted on a correlation matrix, tests that are influenced by certain factors are said to have high factor loadings or to load highly on those factors.

2. It is hard for me not to act aggressively if I am angry with someone.

3. Physical aggression does not help to solve problems, it only makes situations worse.

4. There is nothing wrong with a husband hitting his wife if she has an affair.

5. Physical aggression is often needed to keep things under control.

6. When someone makes me mad, I don't have to use physical aggression. I can think of other ways to express my anger.

<u>The following is the data obtained from 14 adolescents:</u>

	Items					
Person	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0	1	0
3	1	0	1	1	1	0
4	1	1	1	1	1	1
5	1	1	1	1	1	1
6	0	0	1	0	0	0
7	0	0	1	1	1	0
8	1	1	1	1	1	0
9	0	0	0	1	0	0
10	0	1	0	1	0	1
11	1	1	1	0	1	1
12	0	0	1	1	1	1
13	0	0	0	0	0	0
14	0	0	0	0	0	0

Factorial validity of the above scale can be assessed using factor analysis. The primary purpose of conducting factor analysis on the scores of the six-item scale is to find support for the assumption that this six-item scale is measuring a single construct: "adolescents' attitude towards the use of physical aggressive behaviour in daily life". If this assumption is supported, a factor analysis on this set of data should point to a one-factor solution.

In the pages that follow, we will first outline and briefly explain the usage of the major commands and subcommands for SPSS factor procedure. Then, the entire computer program for factor analysis will be shown. This will in turn be followed by detailed discussions on the computer outputs.

SPSS commands for Factor Analysis:

Only commands and subcommands pertinent to the needs and purposes of our present analysis are shown. These represent only a very small proportion of the commands and subcommands for doing factor analysis on SPSS. Please read SPSS User's Guide for other appropriate commands and subcommands in the factor procedure not mentioned in the following discussion.

```
Factor     variables=item1 to item6/
               analysis=item1 to  item6
```

The "variables" subcommand lists the variables[3] to be analyzed in the factor analysis, in this case, all the 6 items will be analyzed. The "analysis" subcommand allows us to perform analyses on subsets of the variables named on the "variables" subcommand. But since we would like to do the factor analysis on all the six items, therefore, items listed under "analysis" subcommand are the same as those in the "variables" subcommand. Actually, the "analysis" subcommand is redundant here and can be omitted from the present program.

There are two major phases in the factor analysis, namely, the factor extraction phase and the rotation phase. Several different strategies are available on each of the two phases in the SPSS factor procedure[4]. For the purpose of the present analysis, the principal components analysis (pc) and the varimax rotation methods will be used. Since these two methods are the defaults, we don't need to write the "extraction=pc" and "rotation=varimax" subcommands into the computer program.

The "variables" and "analysis" subcommands in the above computer program will instruct the computer to give us basically all the information we need for the preesent factor analysis. The computer output will contain the following statistics and matrices: the initial statistics, factor matrix and final statistics obtained from the principal components analysis of the extraction phase; and the rotated factor matrix and factor

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[3] "Variables" can mean a lot of things, including tests, sub-tests, individual items on a scale, etc.

[4] The folowing methods can be picked to carry out the extraction phase in the SPSS factor procedure: principal components analysis (the default); principal axis factoring; maximum likelihood; alpha factoring; image factoring; unweighted least squares; and generalized least squares. In the rotation phase, the following methods are available in the SPSS factor procedure: varimax rotation (the default); equamax rotation; quartimax rotation; direct oblimin rotation; and no rotation.

transformation matrix obtained from the varimax rotation method. However, if we also want to include in the output the correlation matrix of the items, the scree plot, and the factor loading plot for the varimax rotation, we have to write into the computer program two additional subcommands as shown in the following:

```
factor          variables=item1  to  item6/
                print=correlation/
                analysis=item1 to item6/
                plot=eigen rotation (1,2)
```

The "print=correlation" subcommand instructs the computer to print the correlation matrix of the items into the output. The "plot=eigen" subcommand directs the SPSS factor procedure to produce the scree plot, which is a plot of factors versus their eigenvalues. The "plot=rotation (w,2)" subcommand directs the computer to plot the factor loadings of factors 1 and 2 obtained from the varimax rotation method.

Conducting Factor Analysis on the Set of Scores Obtained from 14 Adolescents for the 6 Items Using SPSS

1. SPSS Computer Program for the Analysis

2. SPSS Outputs and Discussions[5]

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The initial part of the output contains a correlating matrix showing the correlation coefficients among the items[6].

Correlation Matrix

		ITEM1	ITEM2	ITEM3	ITEM4	ITEM5	ITEM6
Correlation	ITEM1	1.000	.689	.645	.344	.645	.378
	ITEM2	.689	1.000	.344	.344	.344	.689
	ITEM3	.645	.344	1.000	.417	.708	.344
	ITEM4	.344	.344	.417	1.000	.417	.344
	ITEM5	.645	.344	.708	.417	1.000	.344
	ITEM6	.378	.689	.344	.344	.344	1.000

[5] Discussions and Explanations are in italics. These are not parts of the original computer output.

[6] Please note that the 1-tailed Significance of the Correlation Matrix will also be given if the SPSS for Windows Program is used. However, the SPSS Program used here will not give the 1-tailed Significance.

It is shown in the above correlation matrix that the largest correlation coefficient occurs between item 3 and item 5 (i.e. r = .70833). The second largest correlation coefficient is .68889, which occurs between items 1 and 2 as well as between items 2 and 6. The smallest correlation coefficient is .34427, and all the following 7 pairs of items have this correlation coefficient: items 2 and 3: items 2 and 5; items 4 and 1; items 4 and 2; items 4 and 6; items 6 and 3; and items 6 and 5. The next smallest correlation coefficient is .37778, which occurs between items 1 and 6. Based on the above correlation matrix and using the crude method of "eyeball" factor analysis, it can be suggested that there are possibly two factors. Item 2 and item 6 seem to load on one factor and items 1, 3 and 5 on the other. However, item 2 is also highly correlated with item 1 (r = .68889). Besides, item 4 itself can possibly load on another factor (because it is not highly correlated with any one of the other factors), but since the correlation coefficients between item 4 and each of the items 3 and 5 amount to .41667 which cannot be regarded as particularly low, it can be postulated that item 4 may load on the same factor as items 3 and 5.

However, the above suggestions are based on very crude analyses and are not very conclusive. More sophisticated analyses have to be performed by the SPSS factor procedure. The following section of the output are the matrices and statistics obtained from using the method of the "principal components analysis" in the extraction phase of the factor procedure:

EXTRACTION 1 FOR ANALYSIS 1, PRINCIPAL-COMPONENTS ANALYSIS (PC)

Communalities

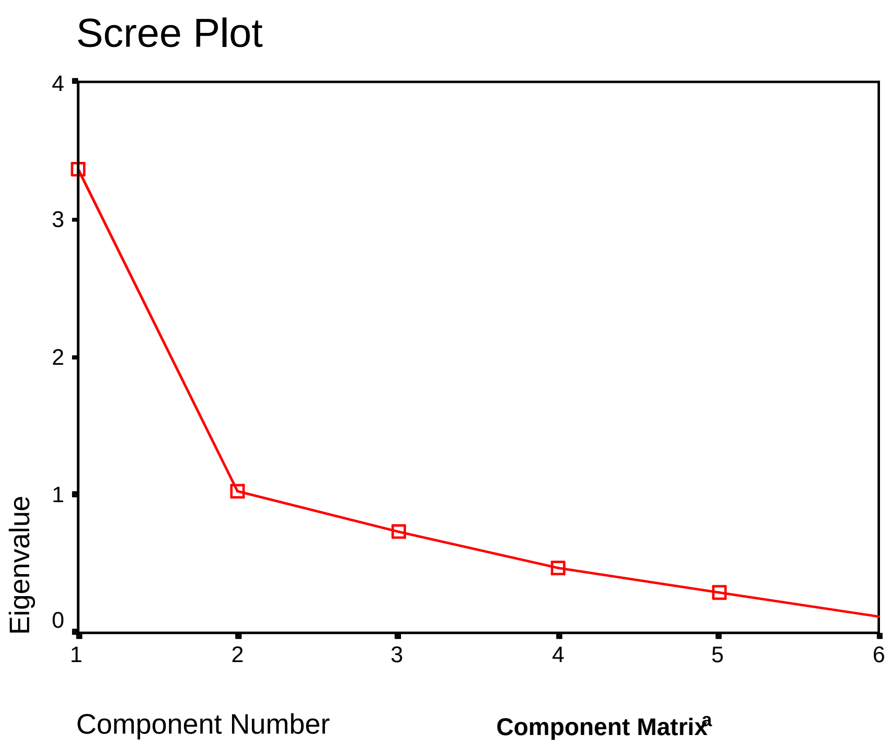
	Initial	Extraction
ITEM1	1.000	.725
ITEM2	1.000	.878
ITEM3	1.000	.805
ITEM4	1.000	.382
ITEM5	1.000	.805
ITEM6	1.000	.790

Extraction Method: Principal Component Analysis.

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.361	56.025	56.025	3.361	56.025	56.025	2.474	41.239	41.239
2	1.024	17.071	73.096	1.024	17.071	73.096	1.911	31.857	73.096
3	.734	12.227	85.323						
4	.471	7.846	93.169						
5	.292	4.861	98.030						
6	.118	1.970	100.000						

Extraction Method: Principal Component Analysis.



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Component Matrix[a]

	Component	
	1	2
ITEM1	.848	-7.50E-02
ITEM2	.762	.545
ITEM3	.785	-.435
ITEM4	.610	-9.62E-02
ITEM5	.785	-.435
ITEM6	.676	.577

Extraction Method: Principal Component Analysis.

a. 2 components extracted.

hat principal components analysis basically does is to transform a set of correlated variables (in this case, each item is a variable) to a set of uncorrelated variables (or principal components). In the context of the present phase of factor analysis, these uncorrelated variables are the factors. In principal components analysis, linear combinations of the observed variables are formed. The first principal component (or factor in this context) is the combination that accounts for the largest amount of variance in the sample. The second principal component accounts for the next largest amount of variance and is uncorrelated with the first. Successive components explain progressibely smaller portions of the total sample variance, and all are uncorrelated with each other.

*The matrix under the heading "**Initial Statistics"** contains the solution of the principal components analysis when all possible factors are included in the solution. It is possible to compute as many principal components as there are variables; in this example, there can be as many as 6 principal components or factors. These "Initial Statistics" basically tell us the amount of variance explained by each factor as well as the percentage of the total variance in the sample attributed to each factor. To make more sense out of these statistics, we need to understand how the total variance of the sample is worked out.*

The total variance is the sum of the variance of each variable (or item in this case). In the principal components analysis, all variables and factors are expressed in standardized form, with a mean of 0 and a standard deviation of 1. Since there are 6 items in this example and each is standardized to have a variance of 1, the total variance is 6 in this example.

The eigenvalue is the amount of variance explained by each factor. The 1^{st} column of the "Initial Statistics" contains 6 eigenvalues, one for each of the possible 6 factors respectively. The sum of these 6 eigenvalues is equal to the total variance of the sample, that is 6. The 6 eigenvalues are arranged in a descending order, with the largest at the top of the column and the lowest at the bottom. Among the 6 factors, factor 1 accounts for the largest amount of variance in the sample, while factor 6 contributes to the smallest amount. For factor 1, the corresponding eigenvalue is 3.36147, which means that out of a total variance of 6, 3.36147 can be attributed to factor 1. It follows naturally that the percentage of variance accounted for by factor 1 is 56.0%, which is obtained from the following computation: 3.36147 divided by 6 times 100%. For factor 6, the eigenvalue and the percentage of variance accounted for are 0.11817 and 2.0% respectively.

It is necessary to briefly introduce the Factor Analysis Model in order to understand the meaning of communality. Under the Factor Analysis Model, the z-scores on variable i are seen as combinations of basically two components, namely, summation of scores on m common factors, and scores on the factor unique to variable i. The Factor Analysis Model is best illustrated by the following equation:

$$z_i = \sum_{k=1}^{m} a_{ik} f_k + u_i \qquad \text{(Equation 1)}$$

where z_i represents z-scores on variable i, a_{ik} represents the loadings of variable i on factor k[7]*, f_k represents scores on common factor k, and u_i represents scores on the factor unique to variable i. A common factor is a factor with which 2 or more variables are correlated and hence contributes to the observed correlations between these variables; it is actually the same as the "factor" which we have consistently been referring to in the above discussion. A unique factor is correlated with only one variable (a unique factor therefore should be uncorrelated with any of the common factors and should also not be correlated with any unique factors for other remaining variable) and hence does not account for correlations between variables*[8]*.*

An important question in a factor analysis is the portion of a variable's variance that is associated with variance on the common factors (i.e., the proportion of the variable's variance that is explained by the common factors). This amount is called communality or the common variance and is calculated by

$$h^2_i = \sum_{k=1}^{m} a^2_{ik} \qquad \text{(Equation 2)}$$

(h^2_i is the communality of variable i)

for uncorrelated factors[9]*. The proportion of a variable's variance associated with variance on its unique factor is called the uniqueness or the unique variance and is calculated by*

$$u^2_i = 1 - h^2_i \qquad \text{(Equation 3)}$$

(u^2_i is the uniqueness of variable i)

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7 Loadings of variables on factors (or factor loadings) will be illustrated and explained in a later section.

8 The theoretical relationship between inter-variable correlations and factor loadings will be shown and explained in a later section.

9 In a later section, an illustration will be given on how communality can be calculated from factor loadings.

Theoretically, $u^2_i = s^2_i + e^2_i$*, where* s^2_i *is the specific variance of a variable (i.e. the portion of a variable's true score unrelated to true score variance on any of the other variables included in the factor analysis), and* e^2_i *is the error variance. Therefore, the total variance (which has been standardized as 1) can be expressed by*

$$\boldsymbol{h^2_i + s^2_i + e^2_i = 1} \qquad \text{(Equation 4)}$$

The communality is usually a number less than 1. In the "Initial Statistics" matrix, all the communalities are 1. This is because all factors are included in this solution. When all factors are included in the solution, all of the variance of each variable is accounted for, and there is no need for a unique factor in the model. The proportion of variance accounted for by the common factors (a total of 6 factors in this case), or the communality of a variable, is therefore 1 for all the variables.

*<u>Determination of the Number of Factors in the Model</u> - However, if all the possible factors are included in the solution, there is nothing gained since there are as many common factors (or simply factors or principal components) as variables. A common criterion used to determine the number of factors to use in the model is that only factors that account for variances greater than 1 (i.e. The eigenvalue is greater than 1) should be included. This is in fact the default criterion in the SPSS factor procedure. The rationale behind this criterion is that factors with a variance less than 1 are no better than a single variable, since each variable has a variance of 1. In the present example, the computer procedure suggests that a model with two factors may be adequate to represent the data (as shown in the "**Factor Matrix"** and the **"Final Statistics"**). It can be seen from the "Final Statistics" that each of these two factors has an eigenvalue greater than 1 (i.e. 3.36147 and 1.02428 respectively) and they together account for over 73% of the total variance of the sample.*

Another method that can be used to decide the number of factors in the model is to inspect the <u>scree plot</u>, a plot instigated by the "plot=eigen" subcommand and produced between the "Initial Statistics" and the "Factor Matrix" in the output. Typically, the plot should show a distinct break between the steep slope of the large factors and the gradual trailing off of the rest of the factors. The gradual trailing off is called the scree, and experimental evidence indicates that the scree begins at the kth factor, where k is the true number of factors. The scree plot in the present output basically suports a 2-factor solution, because the scree begins at the 2nd factor [10].

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[10] There may be controversies on whether factor 2 should be included in the model because its eigenvalue just barely exceeds 1 and it is also likely that "it is already on the scree" in the scree plot. However, since this factor explains over 17% of the total variance in the sample, it will not be unreasonable to include it in the model.

<u>Factor Loadings and Communality</u> - The figures produced in the "Factor Matrix" are the factor loadings. To put it simply, the factor loading of a variable on a factor represents how much weight is assigned to the factor. When the factors are orthogonal (that is, when they are uncorrelated with each other), the factor loadings are also correlations between the factors and the variables. Since the principal components analysis is used to extract factors in this example, the resulting two estimated factors must be orthogonal. Hence, from inspecting the figures in the "Factor Matrix", we can say for example that the correlation between item 2 and Factor 1 is .7624 and that the correlation between item 6 and Factor 2 is .57746, etc.

Since the two resulting factors are orthogonal, the communality shown in the "Final Statistics" can be calculated from the appropriate factor loadings in the "Factor Matrix" using equation 2 already discussed in an earlier section:

$$\mathbf{h}^{\mathbf{2}}_{i} = \sum_{k=1}^{m} a^{2}_{ik}$$

For example, the communality of item 1 can be obtained from adding the squares of its factor loadings on Factor 1 and Factor 2, i.e., $.72545 = (.84843)^2 + (-.07500)^2$. *This communality of item 1 means that 72.545% of the variance of item 1 is explained by the two common factors.*

<u>Relationship between factor loadings and variable inter-correlations</u> - One of the basic assumptions of factor analysis is that the observed correlation between variables is due to the sharing of common factors. Therefore, the correlation between a pair of variables has a very important relationship to the loadings of the two variables on the factors. When the factors are orthogonal, the general equation relating the variable intercorrelations to factor loadings is

$$p_{ij} = \sum_{k=1}^{m} a_{ik}\, a_{jk} \qquad \text{(Equation 5)}$$

where p_{ij} *is the correlation between scores on variables i and j (in this example, it is the correlation between items i and j),* a_{ik} and a_{jk} *are respectively the factor loadings of variable i and j on factor k, and m is the number of factors. When there are two factors like this example, the relationship is:*

$$p_{ij} = a_{i1} a_{j1} + a_{i2} a_{j2}$$

As an illustration, the factor loadings of item 4 and item 6 on factor 1 and factor 2 are used to compute the correlations between item 4 and item 6. p_{46} *is therefore equal to the sum of* $a_{41} a_{61}$ *and* $a_{42} a_{62}$, *which is .35690 (this value is obtained from (.61204) (.67586) + (-.09618) (.57746)). When compared with the observed correlation coefficient between items 4 and 6 (i.e.* r_{46} = .34427) *in the correlation matrix given in the output, there is a difference of 0.01263. This difference is called a residual. For this set of data and analysis, this residual is already the smallest. The biggest residual occurs between items 1 and 4, which is equal to 0.18069 (i.e. the difference between the correlation coefficient of .52496 computed from equation 5 and the observed correlation coefficient of .34427 from the correlation matrix in the output)*[11].

There is a major explanation for the occurrence of residuals (i.e., the differences between correlations among pairs of items calculated from the equation and their corresponding observed correlations): For correlation calculated for a sample, equation 5 will be satisfied exactly only for N - 1 factors, where N is the number of variables. It follows that in this example, equation 5 can be satisfied exactly only in a 5-factor solution. Since the present calculations are based on a 2-factor solution which explains about 73% of the total variance in the sample, the discrepancies between the observed and computed correlations are expected to occur.

Factor loadings shown in the ***Factor Matrix*** *are usually called* <u>*initial or unrotated loadings*</u> *because they are obtained by using a method that permits convenient calculation of the loadings. Typically, researchers do not attempt to interpret these unrotated loadings or this factor matrix. It is because very often the variables and factors in this matrix do not appear correlated in any interpretable pattern and it is usually difficult to identify meaningful factors based on this matrix. The factor matrix of unrotated loadings in this example is not easy to interpret either. All items have very high loadings on factor 1, and if we follow the conventional rule that loadings less than .30 are considered unimportant, 4 items (i.e. items 2, 3, 5 and 6) also have very high loadings on factor 2. Even if we raise the critical level to .50, we still have item 2 and item 6 loading highly on factor 2.*

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[11] If the “print=repr” subcommand is included in the computer program, SPSS factor procedure will produce in the output a matrix containing variable inter-correlations estimated by using equation 5 and the residuals.

The Rotation phase of factor analysis attempts to transform the initial matrix into one that is easier to interpret[12]. The purpose of rotation is to achieve a simple structure. Essentially, the simple structure criteria imply that each variable should have large loadings on as few of the factors as possible (preferably one) and low or zero loadings on the remaining factors. Factor matrix that satisfies the simple structure criteria permits the factors to be differentiated from each other, or to put this in another way, such matrix would allow easier identification of sets of closely related variables.

There are two classes of rotations, orthogonal and oblique. Orthogonal rotations result in uncorrelated factors, whereas oblique rotations result in correlated factors. Both classes of rotations involve finding new axes in a factor loadings plot so that the axes pass closer to clusters of variables. But the new axes must be perpendicular for orthogonal rotations, whereas the new axes of an oblique solution are not perpendicular. Rotation does not affect the goodness of fit of a factor solution, that is, although the factor matrix changes, the communalities and the percentage of total variance explained do not change. Factor loadings resulting from orthogonal solutions satisfy both equations 2 and 5, but if these two equations are applied to factor loadings obtained from oblique solutions, they will be satisfied only when additional terms that take into consideration of the correlations among the factors are added to the equations.

In this example, the varimax rotation method is used. Varimax rotation belongs to the class of orthogonal rotations, thus it will result in uncorrelated factors. The output reproduced below are the results after the varimax method has been applied to the initial factor matrix. It contains a rotated factor matrix, a factor transformation matrix and a factor loadings plot. Since the factor transformation matrix is not particularly relevant to our present discussion, discussions will be mainly focussed on the rotated factor matrix and the factor loadings plot.

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12 The process of transforming the initial matrix into one that is easier to interpret is quite complicated and will not be explained here. Basically, all methods of rotation wil result in a set of transformation equations used to transform the initial factor loadings so that the approximate simple structure (the principle of simple structure will be dealt with later). With reference to the present example, when the items have been plotted on a two-dimensional plot using the initial factor loadings as coordinates with different factors represented by different axes (i.e. factor 1 on the x-axis and factor 2 on the y-axis), rotation phase of factor analysis basically involves finding two new axes, one that passes closer to the first cluster and a second that passes closer to the second cluster. A plot of factor loadings resulting from varimax-rotated solution will be included as an illustration in a later section.

VARIMAX ROTATION 1 FOR EXTRACTION 1 IN ANALYSIS 2 - KAISER NORMALIZATION

Rotated Component Matrix[a]

	Component	
	1	2
ITEM1	.714	.464
ITEM2	.265	.899
ITEM3	.886	.140
ITEM4	.540	.300
ITEM5	.886	.140
ITEM6	.177	.871

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

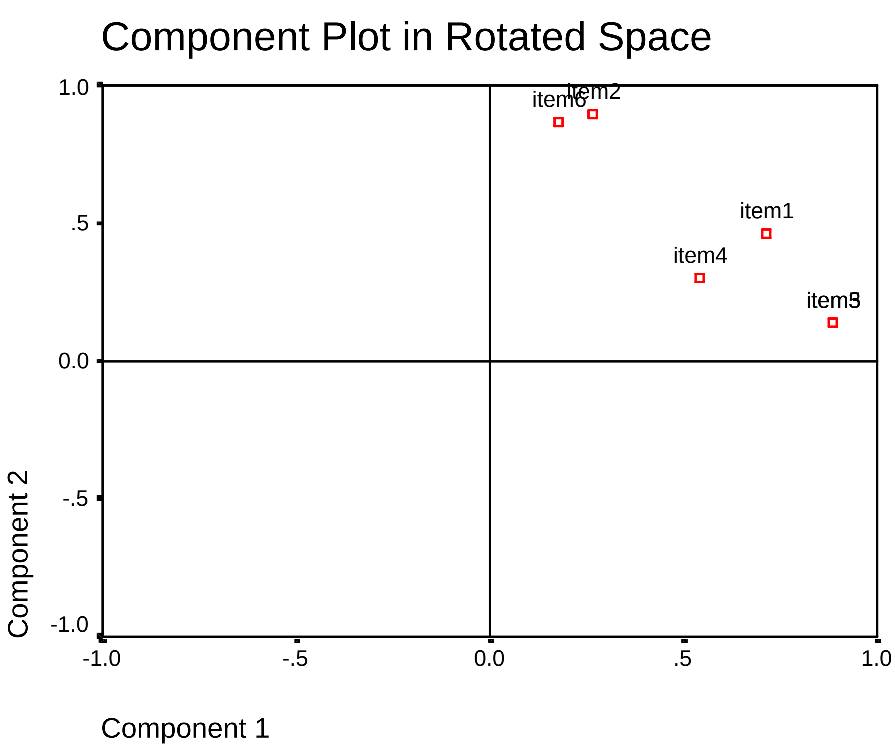
a. Rotation converged in 3 iterations.

FACTOR TRANSFORMATION MATRIX:

Component Transformation Matrix

Component	1	2
1	.788	.616
2	-.616	.788

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.



Component Score Coefficient Matrix

	Component	
	1	2
ITEM1	.244	.098
ITEM2	-.149	.559
ITEM3	.446	-.191
ITEM4	.201	.038
ITEM5	.446	-.191
ITEM6	-.189	.568

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

*The **"Rotated Factor Matrix"** appears to be a little more easy to interpret than the previous "Factor Matrix". With the exception of item 1, all other items have high factor loadings on only one factor. If .5 is employed as the critical value of factor loadings (i.e. factor loadings smaller than .5 are considered unimportant), we can draw the following conclusions from this factor matrix:*

a. *A two-factor solution is adequate to represent the data;*
b. *Items 1, 3, 4, and 5 load primarily on factor 1, while items 2 and 6 load primarily on factor 2.*

The above conclusions seem to match the results of the "eyeball" factor analysis described on page 8. However, these results do not seem to support the original assumption, that is, this 6-item scale measures a single construct: adolescents' attitude towards the use of physical aggressive behaviours in their daily life.

*Another convenient means of examining the success of an orthogonal rotation is to plot the variables using the factor loadings as coordinates on a **factor loadings plot** (this plot is reproduced on page 17 and represents the results of the varimax rotation). The plotted numbers represent the numbers of the variables (or items in this case), e.g. 2 represents item 2. The coordinates of each plotted number correspond to the factor loadings in the rotated factor matrix, with factor 1 represented by the x-axis and factor 2 by the y-axis. The coordinates are also listed under the plot.*

If a rotation has achieved a simple structure, clusters of variables should occur near the ends of the axes and at their intersection. Variables at the end of the axis are those that have high loadings on only that factor. Variables near the origin of the plot have small loadings on both factors. Variables that are not near to the axes are explained by both factors. If a simple structure has been achieved, there should be few, if any, variables with large loadings on more than one factor.

There are 5 plotted numbers in the reproduced factor loadings plot instead of 6. This is because item 3 and item 5 have the same coordinates (i.e. same loadings on factor 1 and same loadings on factor 2), and the computer only prints one of them, i.e. 5, on the plot. Basically most of the plotted numbers are located near the ends of the axes. Item 1 appears to be a little away from the axes because it also has comparatively high loading on factor 2. Nevertheless, the plot indicates that the rotation has achieved a relatively simple structure, with the majority of the plotted numbers located near the ends of the axes.

3. <u>What Conclusions can we Draw from the Factor Analysis on this Set of Data on the 6-item Scale:</u>

Instead of a predicted one-factor solution, a 2 - factor solution is reached by this factor analysis. The original assumption that the 6 - item scale is measuring one construct is not supported by the analysis. A closer look at the 6 items may suggest why a 2 - factor solution is reached. One of the possible explanations is that items 2 and 6 appear to be related to what the respondents would actually behave in conflict situations, while items 3, 4 and 5 (and to a great extent item 1 also) refer to the use of physical aggressive behaviours in general situations when respondents' immediate and actual responses and behaviours are not directly involved or asked. If we examine item 1 which also has moderately high loading on the factor on which items 2 and 6 primarily load (i.e. factor 2), we can find some support for the above explanation. While item 1 may be primarily designed to refer to some general situation when physical aggressive behaviours were used, the portion of item 1 which reads "people won't listen to you unless you get physically aggressive" may unexpectedly invite the respondents to think about how they would actually behave when they are probed with this item. Based on the above factor analysis results and explanations, we can suggest that the 6 item scale may by measuring two constructs or two dimensions of the same construct. Factor 1 may be measuring adolescents' general attitude towards use of physical aggressive behaviours in daily life and factor 2 may in fact be measuring adolescents' predisposition to act aggressively when provoked. Nevertheless, the above explanation may only be one of the many possible explanations for why a two-factor solution appears to be more appropriate.

Reliability analysis on the same set of data suggests that the scale has a very high inter-item reliability coefficient (Cronbach's $\alpha = .8396$). However, results of the present analysis do not offer support to the claim that this scale is a homogeneous instrument. While reliability analysis suggests that item 4 should be the first to go if some items are to be deleted from the scale, this factor analysis also points to the need to look more closely into item 1. In any event, the scale developer has to revise the scale if he/she hopes that only one construct should be measured by the scale. Other methods for construct validation should also be employed to improve the construct validity of this scale.

Two more reminders about effective use of factor analysis:

a. Rule of thumb for the minimum sample size in factor analysis: 100 subjects or 10 times the number of variables (whichever is larger) (thus, the sample size of this example is too small!);

b. Factors are hypothetical constructs and do not have meanings by themselves. Meaningful interpretation of the resulting factors must have to be based on sound theoretical frameworks and unbiased and careful analyses.

Determining Factorial Validity Using SPSS Factor Procedure

Example 2:

The following questionnaire was developed by a researcher as part of an effort to collect participants' satisfaction with a five-week community-based program designed to teach individuals disease prevention and to encourage healthier lifestyles. The questionnaire contained six items. Respondents were asked to respond to each item according to the following scale:

1	2	3	4	5
Strongly Agree	Agree	No Opinion	Disagree	Strongly Agree

The 6 items in the questionnaire were:

1. The goals of the program are clear.
2. I feel comfortable in discussing my plans, concerns and experiences with the group.
3. The materials covered in the program are helpful.
4. The health contract is useful in assisting me to make healthy lifestyle changes.
5. Overall speaking, the group is supportive.
6. Overall, the program is useful in assisting me develop positive changes towards healthy lifestyles.

The following is the data obtained from 10 participants:

Person	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
1	2	3	1	3	4	2
2	1	2	1	1	3	1
3	4	3	4	5	3	3
4	5	3	2	4	3	2
5	2	1	2	2	1	1
6	3	3	1	3	3	1
7	4	5	2	3	4	2
8	2	1	2	2	1	1
9	2	2	2	2	2	2
10	3	4	2	5	4	2

Factorial validity of the above scale can be assessed using factor analysis. The primary purpose of conducting a factor analysis on the scores of the 6 - item questionnaire is to find support for the assumption that this 6 - item questionnaire is measuring a single construct: “participants’ satisfaction with the community-based program”. This assumption is supported if a factor analysis on this set of data points to a one-factor solution.

Conducting Factor Analysis on the Set of Scores Obtained from 10 Respondents for the 6 Items Using SPSS

1. SPSS Computer Program for the Analysis

2. SPSS Outputs and Brief Conclusions[13]

Correlation Matrix

		ITEM1	ITEM2	ITEM3	ITEM4	ITEM5	ITEM6
Correlation	ITEM1	1.000	.607	.496	.746	.366	.589
	ITEM2	.607	1.000	.071	.599	.891	.539
	ITEM3	.496	.071	1.000	.571	-.134	.696
	ITEM4	.746	.599	.571	1.000	.514	.741
	ITEM5	.366	.891	-.134	.514	1.000	.493
	ITEM6	.589	.539	.696	.741	.493	1.000

13 Brief conclusions are in italics. They are not parts of the original computer outputs.

Component Matrix[a]

	Component	
	1	2
ITEM1	.823	.145
ITEM2	.807	-.528
ITEM3	.557	.790
ITEM4	.901	.144
ITEM5	.687	-.689
ITEM6	.866	.244

Extraction Method: Principal Component Analysis.

a. 2 components extracted.

Rotated Component Matrix[a]

	Component	
	1	2
ITEM1	.721	.423
ITEM2	.273	.925
ITEM3	.936	-.241
ITEM4	.780	.475
ITEM5	7.667E-02	.970
ITEM6	.817	.375

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

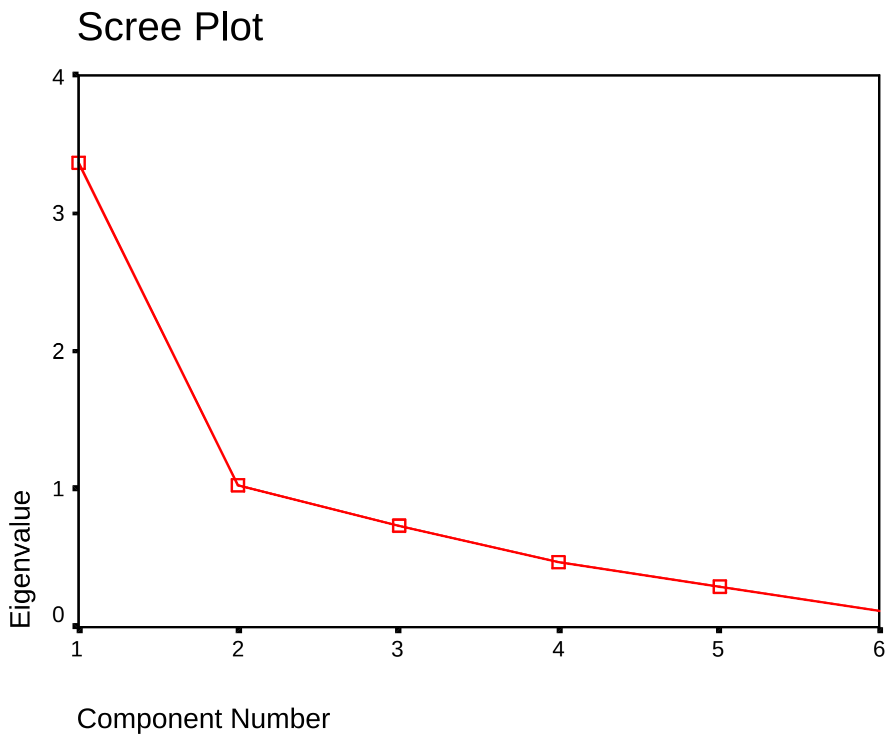
a. Rotation converged in 3 iterations.

3.

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.673	61.221	61.221	3.673	61.221	61.221	2.752	45.865	45.865
2	1.479	24.646	85.867	1.479	24.646	85.867	2.400	40.002	85.867
3	.472	7.874	93.741						
4	.223	3.718	97.460						
5	.122	2.034	99.494						
6	3.038E-02	.506	100.000						

Extraction Method: Principal Component Analysis.



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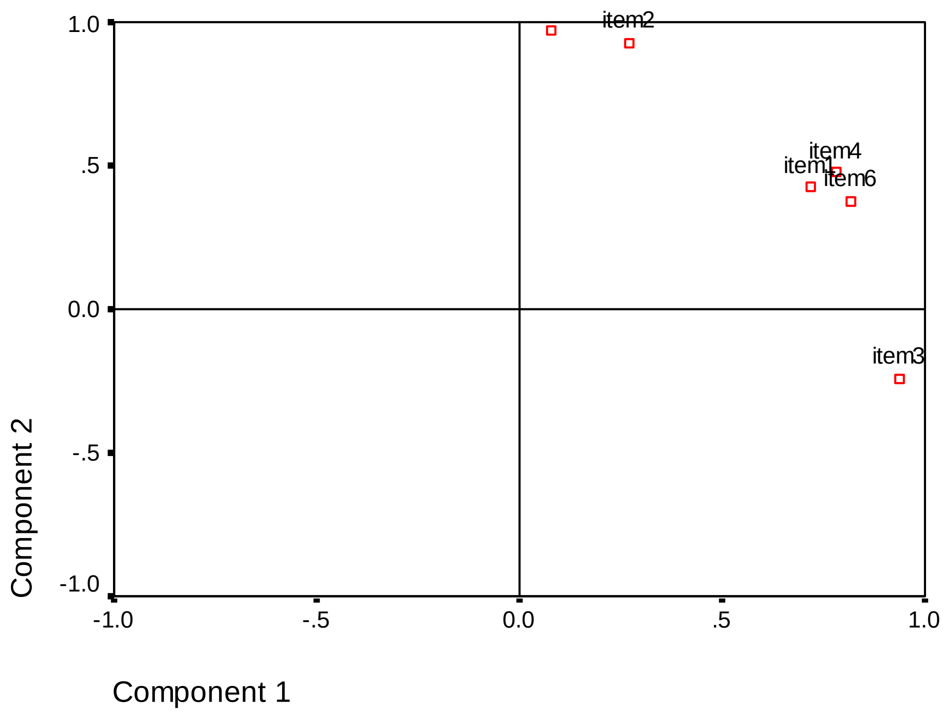
ITEM6	.817	.375

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

Component Plot in Rotated Space



Component Score Coefficient Matrix

	Component	
	1	2
ITEM1	.234	.071
ITEM2	-.064	.415
ITEM3	.462	-.309
ITEM4	.250	.085
ITEM5	-.160	.476
ITEM6	.286	.027

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

3. *<u>Brief Conclusions</u>*

Factor analysis results suggest that a model with two factors may be adequate to represent the data (as shown in the "Final Statistics" of the Principal - Components Analysis and the "scree plot"). These two factors together account for about 86% of the variance of the sample. Based on the results reported in the "Rotated Factor Matrix" and the distribution of the items in the "Factor loadings plot", it can be suggested that items 1, 3, 4 and 6 load primarily on factor 1 and items 2 and 5 load primarily on factor 2. The above results do not lend support to the original assumption that this 6 - item questionnaire is measuring a single construct. A closer examination of the content of the items may suggest a possible explanation for why a two - factor solution appears to be more appropriate: items 1, 3, 4 and 6 may be measuring respondents' satisfaction with the program itself (i.e. its goals, content, etc.); but items 2 and 5 may be more related to respondents' satisfaction with the group (i.e. group atmosphere, cohesion, etc.).

Part Six: Using SPSS for Windows to Conduct Factor Analyses (i.e., tests of factorial validity)

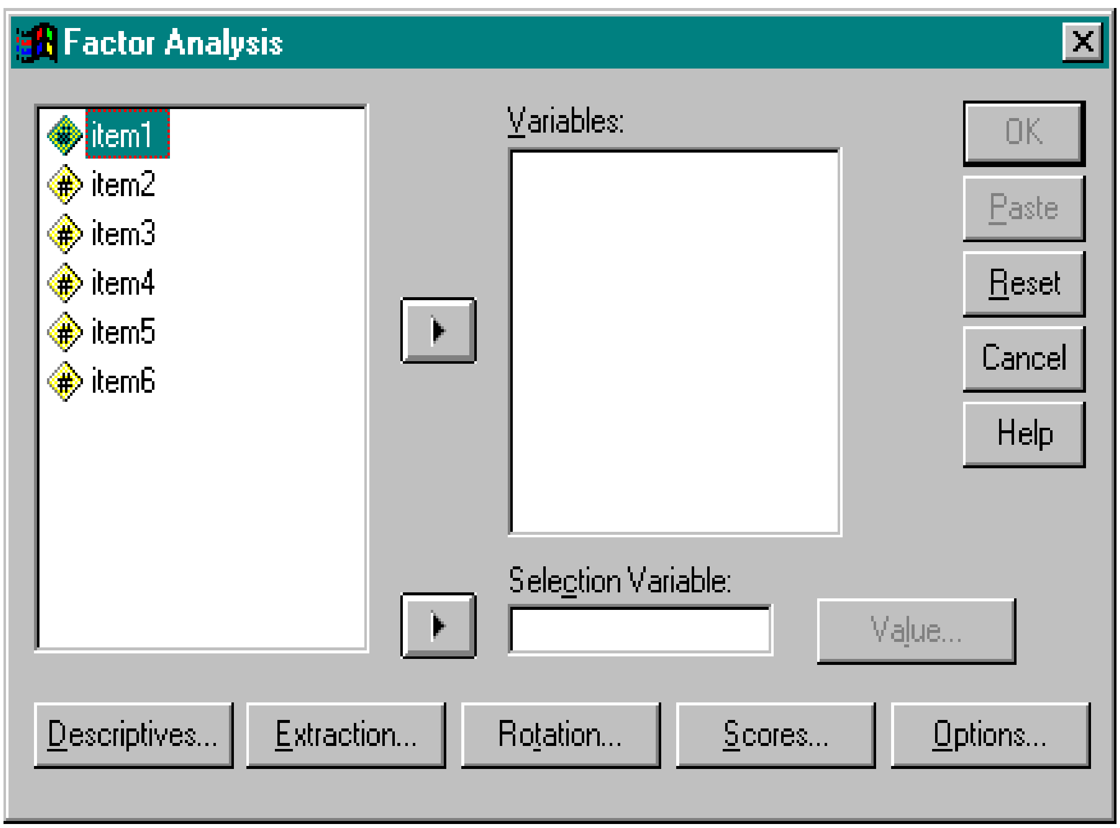
This section will outline the steps necessary for conducting the factor analysis procedure as a test of validity, and should be read in conjunction with Bob Gebotys' "Handout on Validity."

1.1 Specifying the Factor Analysis Procedure

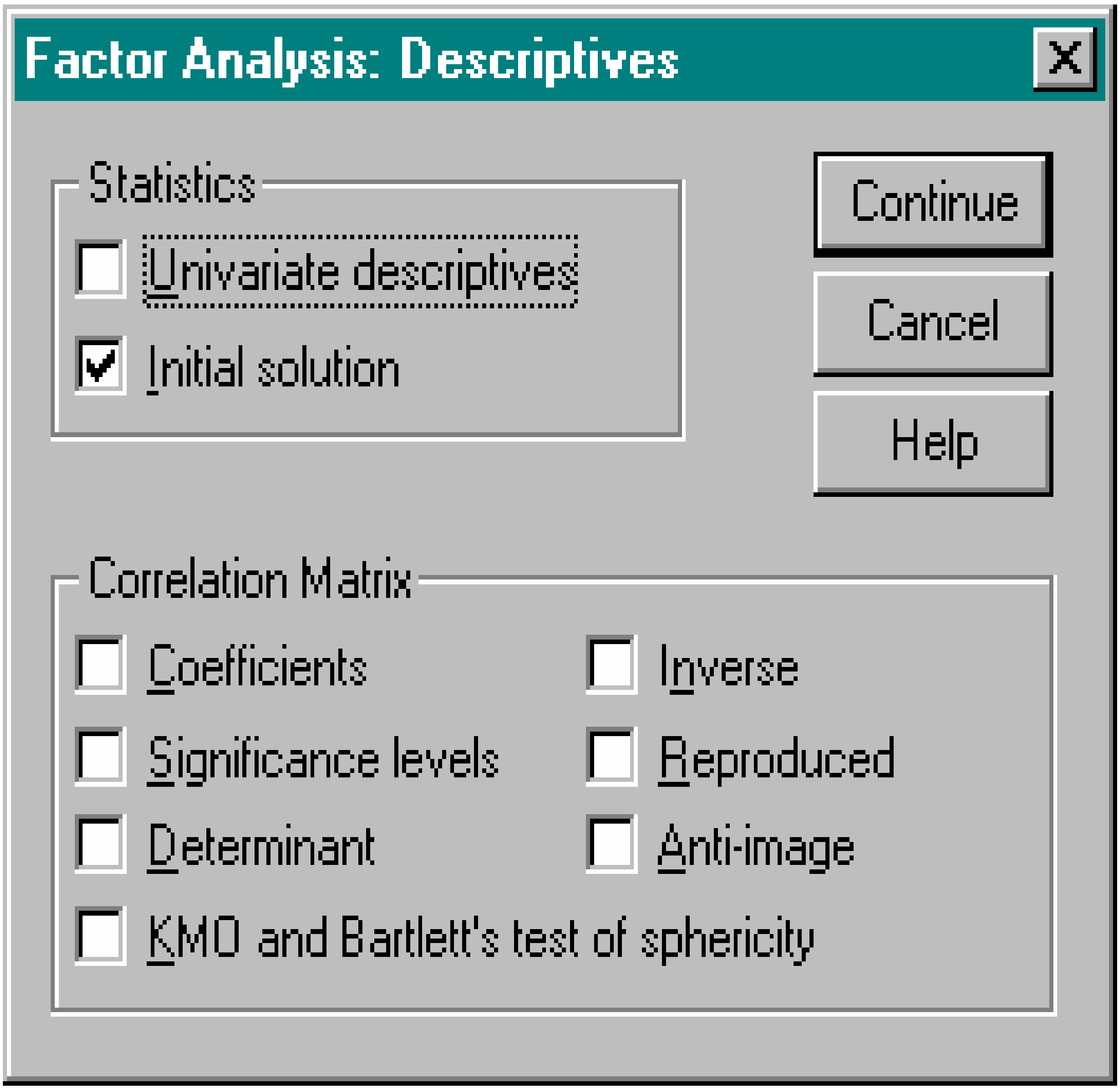
For this analysis it is recommended that one use the data set on adolescent attitudes towards aggression (outlined in Gebotys' "Handout on Validity" p. 5) as used in the previous section on reliability analyses.

The recommended steps are outlined below.

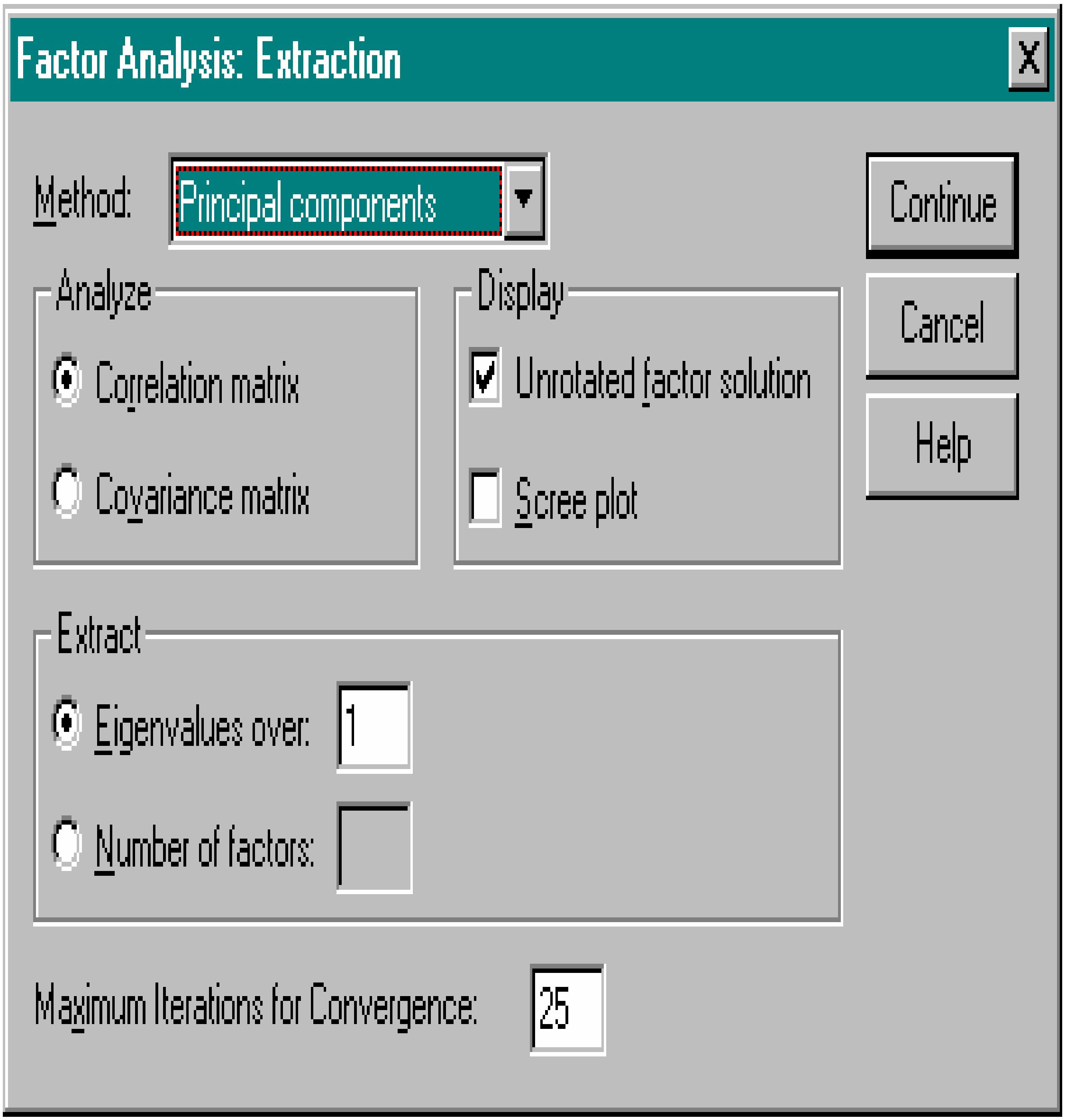
1. Begin by entering the data into a Data Editor window or -- if you have saved the data from the earlier reliability analyses -- retrieve the existing data file.
2. Next, click on **Statistics** on the main menu bar, then **Data Reduction**, followed by **Factor…** This will open a 'Factor Analysis' dialog box similar to the one below.



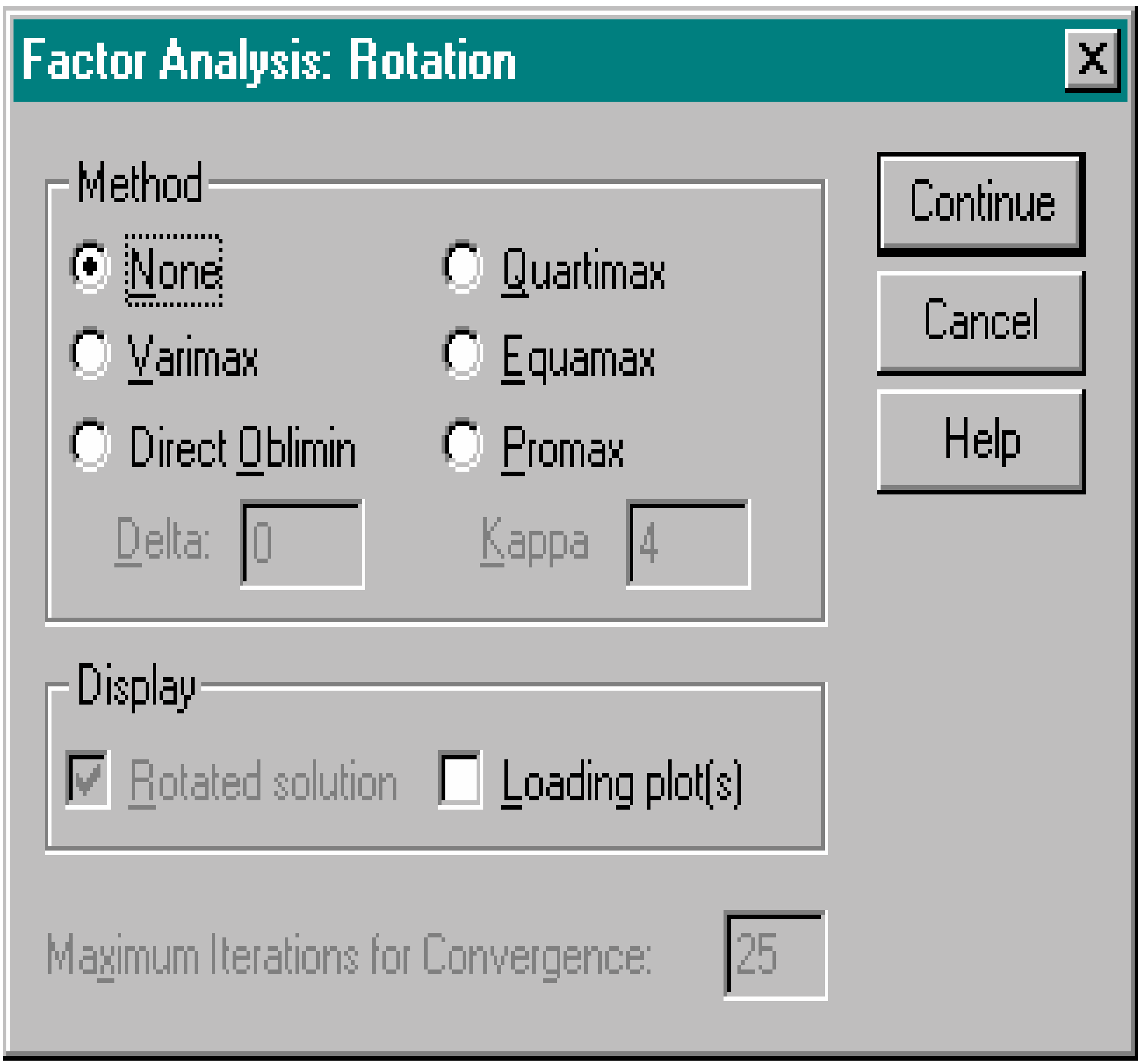
1. Next, select all the variables to be analyzed by clicking on the first item on the list and scrolling your mouse downward until all items are highlighted. Once they are highlighted, click the right facing arrow in the centre of the screen to move the variables to the "**Variables…**" text box.
2. Now, click on the **Descriptives…** pushbutton which will open a 'Factor Analysis: Descriptives' subdialog box similar to the one shown below.



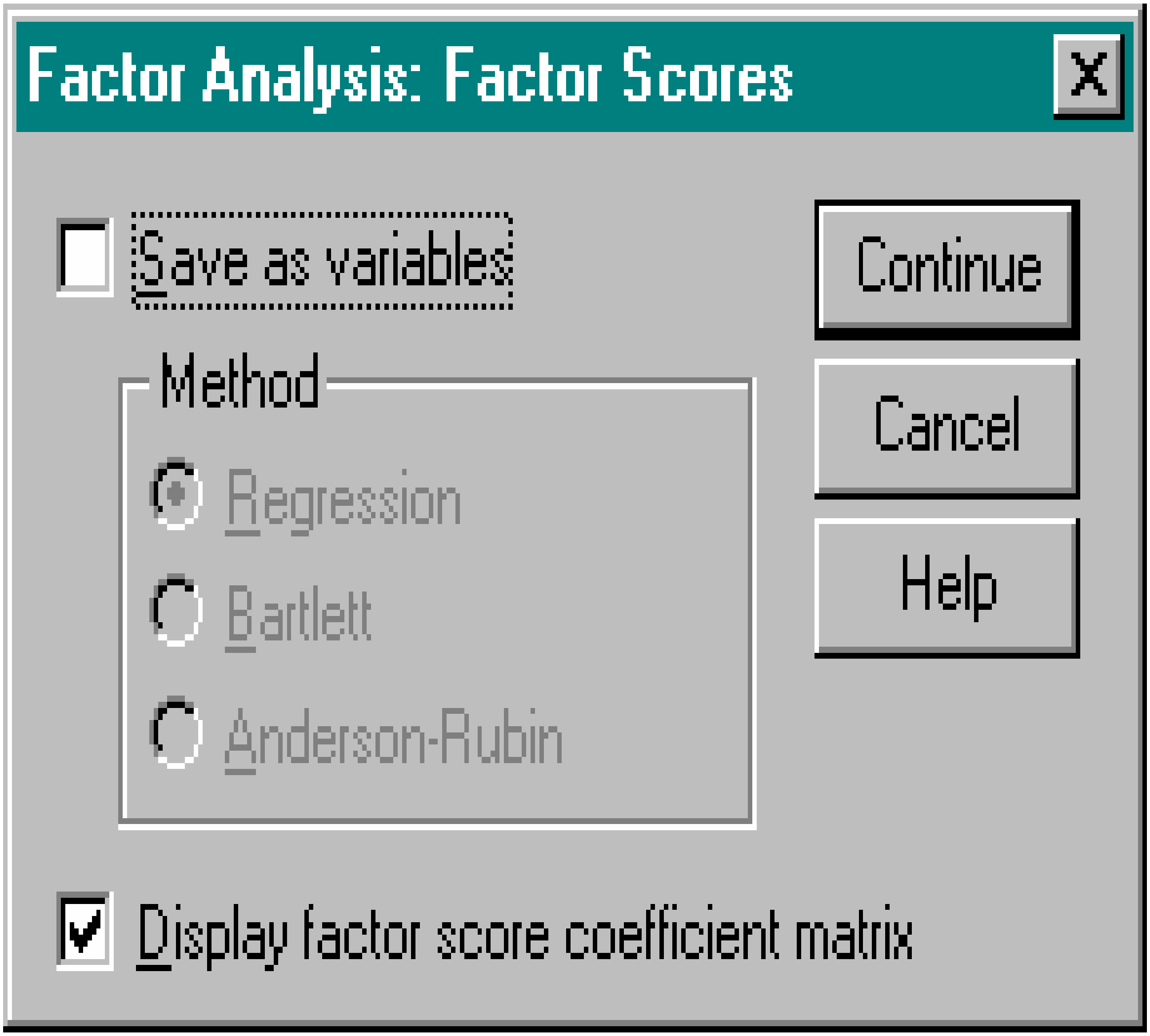
3. Once in the 'Factor Analysis: Descriptives' subdialog box, under the heading 'Correlation Matrix' select '**coefficients'** with a single click on the appropriate check box. Then click the **Continue** command pushbutton, which will return you to the Factor Analysis dialog box.
4. Next, click on the **Extraction…** pushbutton at the bottom of the dialog box, which will open a 'Factor Analysis: Extraction' subdialog box like that shown below.



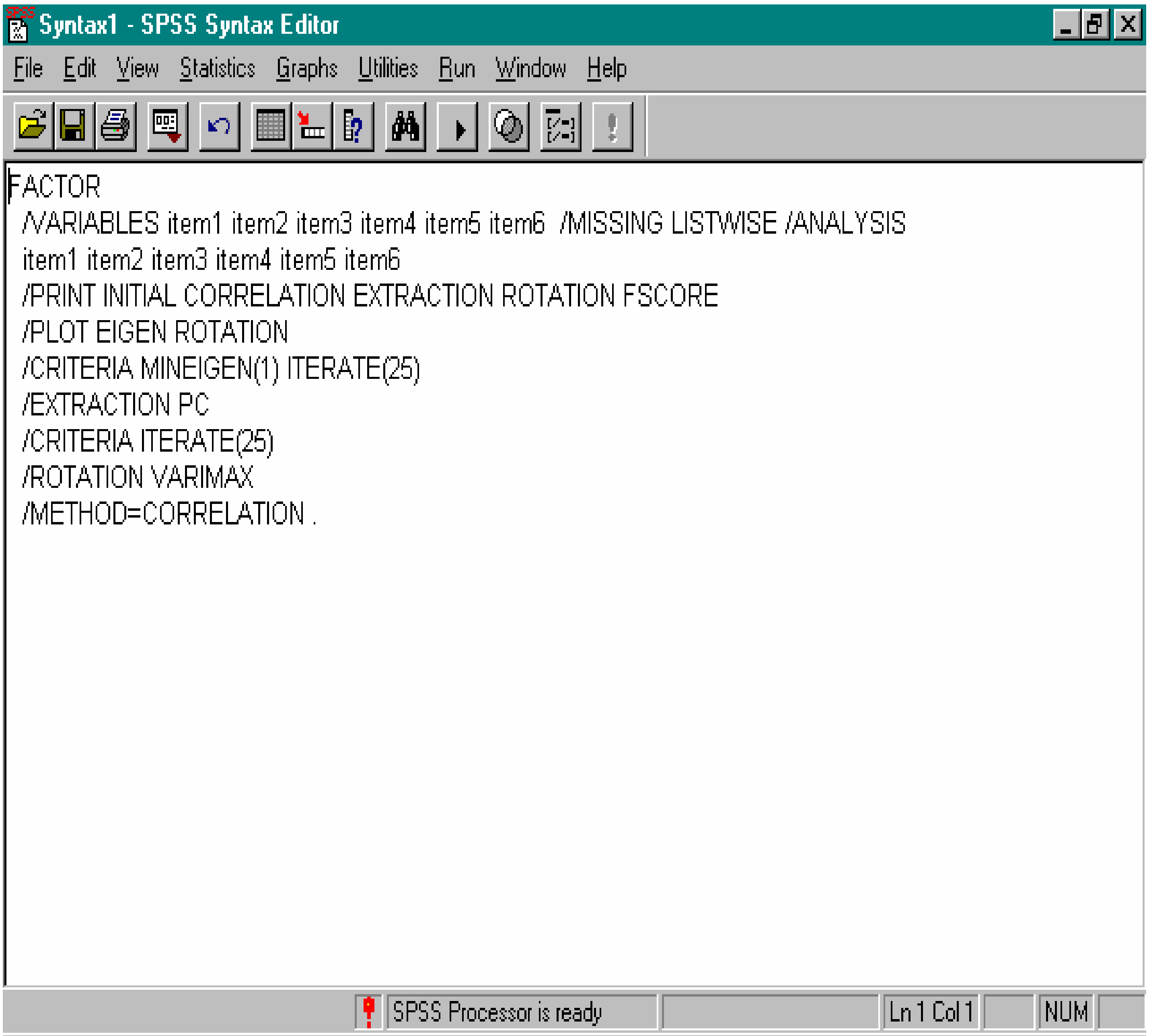
5. If not already selected, under the heading 'Analyze' select "**correlation matrix.**" Under the heading 'Display' select "**unrotated factor matrix**" and "**scree plot**." Under the heading 'Extract' select "**Eigenvalues over 1.**" (Note: the majority of these options will be the '"default" option.) Once these options are selected, click the **Continue** command pushbutton to return you to the Factor Analysis dialog box.
6. Next, click on the **Rotation** pushbutton at the bottom of the dialog box, which will open a 'Factor Analysis: Rotation' subdialog box like the one shown below.



7. Next, under 'Method' select "**V̲arimax**" by clicking on the radio button adjacent to that option. Under 'Display' select "**R̲otated solution**" and "**L̲oading plot(s)**." Then, click the **Continue** command pushbutton to return you to the Factor Analysis dialog box.
8. Click on the '**S̲cores**' pushbutton at the bottom of the dialog box which, will open a 'Factor Analysis: Scores' subdialog box like the one below.



9. Next, select "**Display factor score coefficient matrix**" by clicking on the check box to the left of that option. Then, click the **Continue** command pushbutton to return you to the Factor Analysis dialog box.
10. You have completed all the selections necessary for undertaking this procedure. If you would like to go directly to running the procedure click the **OK** command pushbutton to proceed. If you would like to view the SPSS syntax for this procedure, you may now click the **Paste** command pushbutton, which will open an SPSS Syntax window resembling the one below.



Again, in order to run the Factor Analysis procedure from the SPSS Syntax window, click **Run** on the main menu bar followed by **All**. It is recommended that you save and print the output at this time. The Factor Analysis output should resemble the output on the following pages.