ASSIGNMENT # 5 (Chapter 5)

1.

The scores of students on the ACT college entrance exam in 2001 had a mean of 18.6 and a standard deviation of 5.9. The distribution of scores is only roughly normal.

a) What is the approximate probability that a student randomly chosen from all those scores 21 or higher?

b) Now take and SRS of 50 people who took the test. What are the mean and standard deviation of the sample mean score of these 50 students?

c) What is the approximate probability that the mean score of these students Is 21 or higher?

Answers:

(a) $P(X \geq 21) = P[Z \geq (21.0-18.6)/5.9] = P(Z \geq 0.4068)$

$= 1 - 0.6591 = 0.3409$ OR 34%.

(b) According to the Central Limit Theorem:

$N(\mu, \sigma/\sqrt{n})$; we know that $\mu = \mu_{50} =$ 18.6 ACT score, thus

$\sigma_{50} = \sigma/\sqrt{n} = 5.9/\sqrt{50} = 0.8344$

(c) $P(X \geq 21) = P[Z \geq (21.0-18.6)/0.8344]$

$= P(Z \geq 2.8763) = 1- (0.9980) = 0.002$

OR 2%.

This finding makes sense as the distribution of 50 people is tight to the mean, thus there is less

chance that individual scores fall on extreme tails.

2.

The Gallup Poll once found that about 15% of adults jog. Suppose that in fact the proportion of the population who jog is p = 0.15. What is the probability that the sample proportion of p-hat of the joggers in an SRS of size n=200 lies between 13% and 17%?

Upon reading the question, we identify that this question is dealing with "proportions". Thus, we will be utilizing the equations:

$\mu_{\hat{p}} = p$

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.15(0.85)}{200}} = 0.0252$$

$$P_A(\hat{p} \leq 0.13) = P\left(\frac{\hat{p} - 0.15}{0.0252} \leq \frac{0.13 - 0.15}{0.0252}\right)$$

$$= P(Z \leq -0.7937) = 0.2148$$

$$P_B(\hat{p} \leq 0.17) = P\left(\frac{\hat{p} - 0.15}{0.0252} \leq \frac{0.17 - 0.15}{0.0252}\right)$$

$$= P(Z \leq 0.7937) = 0.7852$$

The probability sample population falls between P_A and P_B, thus $(P_B - P_A)$ = 0.7852 - 0.2148 = 0.5704.

3.

A bottling company uses a filling machine to fill plastic bottles with a popular cola. The bottles are supposed to contain 300 ml. In fact the contents vary according to a normal distribution with mean of 298 ml and standard deviation of 3 ml.

a) What is the probability that an individual bottle contains less than 295 ml?

b) What is the probability that the mean contents of the bottles in a six pack is less than 295 ml?

Answers:

(a) <u>One Bottle</u>

$$P(X < 295) = P\left[\left(\frac{X - 298}{3}\right) < \left(\frac{295 - 298}{3}\right)\right]$$

$$= P(Z < -1) = 0.1587 \quad \text{OR} \quad 15\%$$

(b) <u>Six Pack</u>

$\sigma_6 = \sigma/\sqrt{n} = 3/\sqrt{6} = 1.2247$

$$P(X < 295) = P\left[\left(\frac{X - 298}{1.2247}\right) < \left(\frac{295 - 298}{1.2247}\right)\right]$$

$$= P(Z < -2.4496) = 0.0071 \quad \text{OR} \quad 0.7\%$$