BASIC MATH REVIEW

1. Notation

Parentheses specify control and order of computations. Everything inside the parenthesis is calculated first.

$(2+7)x3 = 9x3 = 27$

Changing the placement of the parenthesis changes the order of the calculations. For example:

$2+(7x3) = 2+21 = 23$

Table 1 - Definitions

Symbol	Meaning	Example
+	Addition	$3+2 = 5$
-	Subtraction	$7-4 = 3$
x, ()	Multiplication	$2x3 = 6$, $2(3) = 6$
÷, /	Division	$4÷2=2$, $4/2 = 2$, $\frac{4}{2}$
>	Greater than	$6>3$
<	Less than	$2<7$
≠	Not equal to	$4≠3$

2. Proportions and Percentages

A proportion is part of a whole. It can be expressed as a fraction, decimal or percentage. For example, of 80 people only 6 are men. Proportion of class that are men can be expressed as a fraction

$$\text{fraction} = \frac{6}{80}$$

or as a decimal value,

decimal = .075

or as a percentage,

percentage = 7.5%

In a fraction the bottom part (the denominator) indicates the number of equal pieces into which the whole is split. For example 2 pieces

If the denominator has a larger value, say 4, the each piece of the whole pie is smaller

The value of the top of the fraction (the numerator) indicates how many pieces of the whole we have. Thus, a fraction $\frac{2}{3}$ means the whole is split evenly into three pieces and that two of them are being considered.

A fraction is a short way of stating a proportion 'two out of three' is equivalent to $\frac{2}{3}$

To convert the decimal to a percentage, simply multiply by 100 and place a percent (%) after the answer.

.66 x 100 = 66%

Consider coins - a dime is one-tenth ($\frac{1}{10}$) of a dollar or .10 as a decimal or 10% as a percentage.

	Dime	Quarter	50 Cents	75 Cents
Fraction	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
Decimal	.10	.25	.50	.75
Percentage	10%	25%	50%	75%

3. Fractions

Equivalent Fractions

The same proportional value can be expressed by many equivalent fractions.

$\frac{1}{2} = \frac{2}{4} = \frac{10}{20} = \frac{60}{120}$

To create equivalent fractions, multiply the numerator and denominator by the same value.

$\frac{1}{4} = \frac{5}{20}$

Multiply both top and bottom by 5.

You can use division to reduce the fraction to a simpler form.

$\frac{5}{10} = \frac{1}{2}$

Both the numerator and denominator have to be divided by 5.

You can find specific equivalent fractions. Find a fraction with denominator 50 and is equivalent to $\frac{2}{5}$

$\frac{2}{5} = \frac{?}{50}$

Notice the denominator of the original fraction must be multiplied by 10 to produce the desired denominator. Therefore, multiply both top and bottom by 10 and obtain:

$\frac{2 \text{ x } 10}{5 \text{ x } 10} = \frac{20}{50}$

Multiplying Fractions

First multiply the numerators and then the denominators.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

Dividing Fractions

To divide one fraction by another you invert the second and then multiply.

$$\frac{1}{3} \div \frac{1}{6} = \frac{1}{3} \times \frac{6}{1} = \frac{1 \times 6}{3 \times 1} = \frac{6}{3}$$

Adding and Subtracting Fractions

Fractions must have the same denominators before you can add or subtract them. If they have a common denominator, you add or subtract only the numerator:

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

The product of the two denominators will always work as a common denominator (although it may not be the lowest).

$$\frac{1}{2} + \frac{2}{20} = ?$$

Use 2x20=40 on the common denominator. The equivalent fraction of each is:

$\frac{1}{2} = \frac{20}{40}$ and $\frac{2}{20} = \frac{4}{40}$

Now we can add

$$\frac{20}{40} + \frac{4}{40} = \frac{24}{40}$$

Comparing Fractions

If they have the same denominator, the larger fraction is the one with the larger numerator:

$$\frac{6}{10} > \frac{2}{10}$$

The denominator is the same so the whole has pieces of the same size. Therefore, 6 pieces are greater than 2.

If there are different denominators, first covert to a common denominator:

$\frac{2}{5}$ and $\frac{3}{10}$

Multiply $\frac{2}{5}$ by 2 to obtain a value in 10ths.

$$\frac{2}{5} = \frac{2x2}{5x2} = \frac{4}{10}$$

Now a comparison can be made:

$$\frac{4}{10} > \frac{3}{10}$$

Therefore

$$\frac{2}{5} > \frac{3}{10}$$

4. Decimals

Like a fraction, a decimal represents part of a whole. The first decimal place to right of decimal point indicates tenths:

$$.1 = \frac{1}{10} \qquad .7 = \frac{7}{10}$$

The next decimal place represents $\frac{1}{100}$, the next $\frac{1}{1000}$, etc. To change to a fraction just use the number without the decimal point for the numerator - use the denominator that the last decimal place (on the right) represents.

$$.85 = \frac{85}{100}$$

$$.05 = \frac{5}{100}$$

$$.001 = \frac{1}{1000}$$

Adding/Subtracting

Keep the decimals in proper columns.

.05	3.264
+1.264	-.82
1.314	2.444

Multiplying Decimals

Multiply the two numbers, ignoring the decimal. Position the decimal point in the answer so that the number of digits to the right of the decimal point is equal to the total number of decimal places in the two numbers being multiplied.

.25 (two decimal places)
x .005 (three decimal places)
125
00
00
.00125 (five decimal places)

Dividing

.25 ÷ 1.6 is equivalent to $\frac{.25}{1.6}$

Multiply both numerator and denominator to remove the decimal using the same value:

$$\frac{.25}{1.6} = \frac{.25x100}{1.6x100} = \frac{25}{160}$$

5. Percentages

Converting a Percentage

For conversion to a fraction, place the number in the numerator and use 100 for the denominator

$$37\% = \frac{37}{100} \qquad 5\% = \frac{5}{100}$$

For conversion to a decimal, divide by 100 or move the decimal two points to the left:

95 % = 95. = .95

97.5% = 97.5 = .975

5% = 5. = .05

Multiplying

What is 45% of 60? or
45% x 60 = ?

First convert 45% to a decimal: .45 x 60 = 27

Questions

1. Convert 6/15 to a decimal
2. Convert 3/5 to a percentage
3. 3/8 = ?/24
4. 7/12 = ?/24
5. 2/7 = ?/14
6. 1/6 x 7/10
7. 7/8 - 1/2
8. 9/10 ÷ 2/3
9. 7/12 + 1/3
10. Convert the decimals to fractions
    a) .99 b) .5 c) .005
11. 2.61 x .05 =
12. 1.4 ÷ .02 =
13. What is 5% of 100?

6. Negative Numbers

Negative numbers represent values less than zero. Note a number without a sign (+ or -) is assumed positive.

Adding: When adding numbers that include negative values, interpret the negative sign as subtraction:

3+(-2)+5 = 3-2+5=6

You can also put all the negative numbers together in a large sum and positive together in the sum and then take away the negative from the positive:

(-4)+5+3(-1)+8(-5)+2+1

positive sum = 19 negative sum = 10

Answer: 19-10=9

Subtracting: To subtract a negative number change it to positive and add:

$5-(-2)=5+2=7$

Multiplying and Dividing: Numbers have the same sign the result is positive. When numbers have different sign the result is negative.

$3x(-4)= -12$
$-2x(-3)= +6$
$-8\div 2 = -4$
$2x-4= -8$
$4\div(-2)= -2$
$-2x-2= 4$

Questions

1. $3+(-4)+6+3+(-4)+(-2)$
2. $6-(-4)+2-(-3)-(-1)$
3. $3+3+4+(-6)+(-3)-(-4)$
4. $9x(-2)$
5. $-6x(-3)$
6. $-6x(-2)x(-3)$
7. $-14\div(-2)$
8. $-20\div(-4)$

7. Solving Equations

An equation is a statement that two quantities are identical:

$6=4+2$

Sometimes an equation will contain an unknown (or variable) quantity that is identified with a symbol.

$6=4+x$

Find the value of x that makes the statement true - $x=2$ makes the equation true. Called solving the equation.

* want x (the unknown) alone on one side of the equation.
Equation remains same if both sides are treated exactly the same.

Examples:

1. $x+4=9$
   subtract 4 from both sides to isolate x
   $x+4-4=9-4$
   $x=5$

2. $x-6=4$
you add 6 to both sides to isolate x
$x-6+6=4+6$
$x=10$

3. $3x=12$
want to remove 3 multiplying x. Therefore, divide by 3:

$$\frac{3x}{3} = \frac{12}{3}$$

$x=4$

4. $\frac{x}{4} = 3$

want to remove 4 dividing x. Therefore, multiply by 4

$$4(\frac{x}{4})=4(3)$$

$x=12$

5. $2x + 5 = 19$
$2x + 5-5 = 19-5$ (remove $+5$ by subtracting 5)
$2x = 14$
$\frac{2x}{2} = \frac{14}{2}$ (remove 2 by dividing by 2)
$x = 7$

6. $$\frac{x+4}{8}=2$$

(remove the 8 by multiplying both sides by 8)

$$8(\frac{x+4}{8})=8(2)$$

$x + 4 = 16$
remove $+4$ by subtracting 4

$x+4-4 = 16-4$
$x = 12$

Questions

1. $3x = 21$
2. $x + 4 = 9$
3. $x - 4 = 16$
4. $5x - 2 = 10$

5. $\frac{x}{10} = 2$
6. $\frac{x+4}{10} = 2$
7. $x - 2 = 5$
8. $\frac{x}{10} = -2$
9. $\frac{3x}{15} = 4$
10. $\frac{x}{2} + 1 = 4$

8. Exponents and Square Roots

Exponential notation is a simple notation used whenever a number is multiplied by itself.

8^4 ← exponent
↕
base

Exponent tells how many times the base is multiplied by itself.

8^4 = 8x8x8x8 (8 raised to the 4th power)
7^2 = 7x7 (7 squared)
2^3 = 2x2x2 (2 cubed)

Note: Any number raised to the first power equals itself.

$5^1 = 5$

Any number (except zero) raised to the zero power equals 1.

$2^0 = 1$

The exponent applies only to the base in front of it.

$xy^3 = xyyy$
$a^2b^2 = aabb$

For a negative number raised to a power, the result in positive for exponents that are even and negative for exponents that are odd.

$-3^3 = -3(-3)(-3)$
$= 9(-3)$
$= -27$

$-2^4 = -2(-2)(-2)(-2)$
$= 4(-2)(-2)$
$= -8(-2)$
$= 16$

Exponents and Parenthesis

If an exponent is outside parenthesis, then the computations within are done first:

$(2+3)^2 = 5^2 = 25$

Note: $2^2 + 3^2 = 4 + 9 = 13$

Note: $\frac{a^2}{b^2}$ can be written as $\left(\frac{a}{b}\right)^2$

$$\frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2$$

$$\frac{4}{9} = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$$

$$\frac{4}{9} = \frac{4}{9}$$

Square Roots

The square root of a value equals a number that when multiplied by itself gives the original value. The square root symbol, √ , is called a radical.

$\sqrt{9} = 3$

$\sqrt{a^2} = a$

$\sqrt{3^2} = \sqrt{9} = 3$

$(\sqrt{a})^2 = a$

$(\sqrt{25})^2 = (5)^2 = 25$

$\sqrt{3+6} = \sqrt{9} = 3$

If the numerator and denominator of a fraction each have a radical, then place the entire fraction under a single radical.

$$\frac{\sqrt{90}}{\sqrt{10}} = \sqrt{\frac{90}{10}} = \sqrt{9} = 3$$

$\sqrt{9} \; x \; \sqrt{16} = \sqrt{9x16} = \sqrt{144} = 12$

Questions

1. -4^3
2. $(2+4)^3$
3. $a^3b^1 \quad a=3 \quad b=10$
4. $a^4b^3 \quad a=2 \quad b=3$
5. $(ab)^2 \quad a=2 \quad b=4$
6. $a^2+b^2 \quad a=2 \quad b=4$
7. $\sqrt{6+3}$
8. $(\sqrt{9})^2$

Basic Mathematics Review

1. Convert $\frac{7}{20}$ to a decimal.
2. Express $\frac{9}{25}$ as a percentage.
3. Convert .91 to a fraction.
4. Express .0031 as a fraction.
5. Next to each set of fractions, write "true" if they are equivalent and "false" if they are not:

   a. $\frac{4}{1000} = \frac{2}{100}$ _____

   b. $\frac{5}{6} = \frac{52}{62}$ _____

   c. $\frac{1}{8} = \frac{7}{56}$ _____

6. Perform the following calculations

   a. $\frac{4}{5} \; x \; \frac{2}{3} = ?$

   b. $\frac{7}{9} \div \frac{2}{3} = ?$

   c. $\frac{3}{8} + \frac{1}{5} = ?$

   d. $\frac{5}{18} - \frac{1}{6} = ?$

7. $2.51 \times .017 = ?$
8. $3.88 \times .0002 = ?$
9. $3.17 + 17.0132 = ?$
10. $5.55 + 10.7 + .711 + 3.33 + .031 = ?$
11. $2.04 \div .2 = ?$
12. $.36 \div .4 = ?$
13. $5 + 3 - 6 - 4 + 3 = ?$
14. $9 - (-1) - 17 + 3 - (-4) + 5 = ?$
15. $5 + 3 - (-8) - (-1) + (-3) - 4 + 10 = ?$
16. $8 \times (-3) = ?$
17. $-22 \div (-2) = ?$
18. $-2(-4) \times (-3) = ?$
19. $84 \div (-4) = ?$

Solve the equations in Problems 20-27 for X.

20. $X - 7 = -2$
21. $9 = X + 3$
22. $\frac{X}{4} = 11$
23. $-3 = \frac{X}{3}$
24. $\frac{X+3}{5} = 2$
25. $\frac{X+1}{3} = -8$
26. $6X - 1 = 11$
27. $2X + 3 = -11$
28. $.5^2 = ?$
29. $-5^3 = ?$
30. If $a = 4$ and $b = 3$, then $a^2 + b^4 = ?$
31. If $a = -1$ and $b = 4$, then $(a + b)^2 = ?$
32. If $a = -1$ and $b = 5$, then $ab^2 = ?$
33. $\frac{18}{\sqrt{4}} = ?$
34. $\sqrt{\frac{20}{5}} = ?$