Some "Numerology" for the Moving Surfaces Implicitization Technique with Gröbner Blending Functions

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March 21, 2008

Abstract

The derivative ideal I' of a bidegree $m \times n$ surface parametrization

$$X(s,t) = \frac{x(s,t)}{w(s,t)}, \quad Y(s,t) = \frac{y(s,t)}{w(s,t)}, \quad Z(s,t) = \frac{z(s,t)}{w(s,t)} \tag{1}$$

is the ideal generated by the parametric polynomials and their partial derivatives; that is,

$$I' = \langle x, y, z, w, x_s, y_s, z_s, w_s, x_t, y_t, z_t, w_t \rangle. \tag{2}$$

Polynomials in the derivative ideal of degree at most m' in s and n' in t form a linear subspace $I'_{m',n'} \subseteq I'$. A basis B of $I'_{m',n'}$ can be obtained from a Gröbner basis G of I'. Moving planes and moving quadrics that follow the surface can then be found with members of B as blending functions. Zheng et al show that such moving planes and quadrics effectively implicitize the surface parametrization in the presence of base points. They also give counting formulas to predict the numbers of blending functions, moving planes, and moving quadrics if the multiplicities of the base points are known. Using these counting formulas, the paper studies the existence of implicitization moving planes and moving quadrics and the characteristics of the blending functions for various surfaces schemes. The schemes investigated include total degree parametrization, surfaces of revolution, and corner-cut surfaces. These schemes introduce a priori known base points structurally whose implicitization effects are nicely revealed by the counting formulas. For example, a total degree n surface parametrization can be treated as a bidegree $n \times n$ parametrization with a triangular base point of multiplicity n. With $m' = n' = n - 1$, there are $\frac{n^2+n}{2}$ blending functions, n moving planes and $\frac{n^2-n}{2}$ moving quadrics that follow the surface; together they produce the degree n^2 implicit equation.