"MACAULAY INVERSE SYSTEMS REVISITED WITH APPLICATION TO CONTROL IDENTIFIABILITY"

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Since its original publication in 1916 under the title "The algebraic theory of modular systems", the book by F.S. Macaulay has attracted a lot of scientists with a view towards pure matematics (D. Eisenbud,...) or applications to control theory (U. Oberst,...). However, a careful examination of the quotations clearly shows that people had only a look to the first three chapters respectively dealing with the resultant, the resolvent and the general properties of modules but mostly did not look at the last chapter dealing with the so-called "inverse system" or, to be fair, only to the particular example of finite dimensional vector spaces on the residue field of a local ring.

The basic intuitive idea is the well known parallel existing between ideals in polynomial rings and systems of PD equations in one unknown with constant coefficients. Accordingly, it becomes evident that Grobner bases should be everywhere ... under the condition to understand what is inside (we invite the reader to have a look at the book in order to discover the difficulty of such a task !).

As a first step done in the last Cambridge paperback edition, a kind of glossary has been exhibited between Macaulay language/definitions and modern corresponding concepts. However, nothing has been done for the last (and most difficult !) chapter and the purpose of this lecture will be first to correct this gap.

Roughly speaking (and though striking it could be !) this chapter is based on the (correct) descripton of "systems" in the sense given by the american school of PD equations in the seventies (H. Goldschmidt, D.C. Spencer, D.G. Quillen,...) and we shall prove that the heart of the chapter is "the use of the Spencer operator on sections" (NOT on solutions, despite formal power series are used !). Of course, this result, (not known to our knowledge) will give by itself quite a new insight on the chapter.

The next idea will be to use modern "algeraic analysis" also pioneered in the seventies (M. Kashiwara, B. Malgrange, V.P. Palamodov,...) in order to extend these ideas to general differential modules and systems of PD equations. Again, we shall prove that the use of Grobner bases is central ... but well hidden as we shll prove that ONE CANNOT AVOID Spencer cohomology, involution and delta-regular coordinates in order to understand in an INTRINSIC WAY the LOCAL COMPUTATIONS done by Macaulay.

Hints will finally be given to generalize these results to the variable coefficients case and the corresponding control identifiability problem. Many explicit examples will be provided for illustrating the main results.