Towards a classification of projective varieties by their degree

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Let $X \subset \mathbb{P}^r_K$ denote an irreducible projective variety over an algebraically closed field. There is a well-known inequality $\deg X \geq \operatorname{codim} X + 1$, where $\deg X$ resp. $\operatorname{codim} X$ denotes the degree resp. the codimension of X. In the case the equality holds the variety X is called a variety of minimal degree. Beginning at least with Castelnuovo much is known about the fine structure of X. For instance, a variety of minimal degree is arithmetically Cohen-Macaulay and its minimal free resolution is linear.

Here we investigate the situation of $\deg X = \operatorname{codim} X + 2$ and $\deg X = \operatorname{codim} X + 3$. The later mainly in the case of X a curve resp. a surface. This is done with elimination techniques based on the theory of Gröbner bases. There are results about the minimal free resolutions and about intrinsic properties of X described by data of a certain type of projections. As additional features we discuss secant varieties, join varieties and Veronese embeddings of projective varieties. To this end we use some SINGULAR-procedures for a discussion of conjectures and counter examples. Part of the research was done jointly with Markus Brodmann (University of Zürich).

In a final part of our talk we will present an interactive computer program for the visualization of the real part of some surface in affine three space. The program is based on the recent development of graphics processing units and shader programming. It provides an interactive visualization of singular surfaces in realtime.

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