

CP400N: Introduction to Parallel Programming: Term Project: Julia Sets Computation Using High Performance Computing

by Nicholas Azar, Dalibor Dvorski, Gabriel St-Onge Carle for Dr. Ilias S. Kotsireas



Department of Physics & Computer Science
Wilfrid Laurier University



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Introduction

In the early twentieth century, Gaston Maurice Julia studied the iteration of polynomials and rational functions. Interesting results may be observed by iterating x in some function $f(x)$. If the iteration of functions is extended to functions containing complex numbers, then we may define Julia set as being generated by the function $Q_c(z) = z^2 + c$, where c is a complex number constant of the form $a + bi$, and where a and b are real numbers. From this it is clear that there are infinite Julia sets, where each is created by defining some value for c . When the resulting geometric points are represented graphically the results are images of great interest. Gaston Maurice Julia studied such sets and it is for his success in this area of mathematics that they hold his surname. [1, 2, 3]

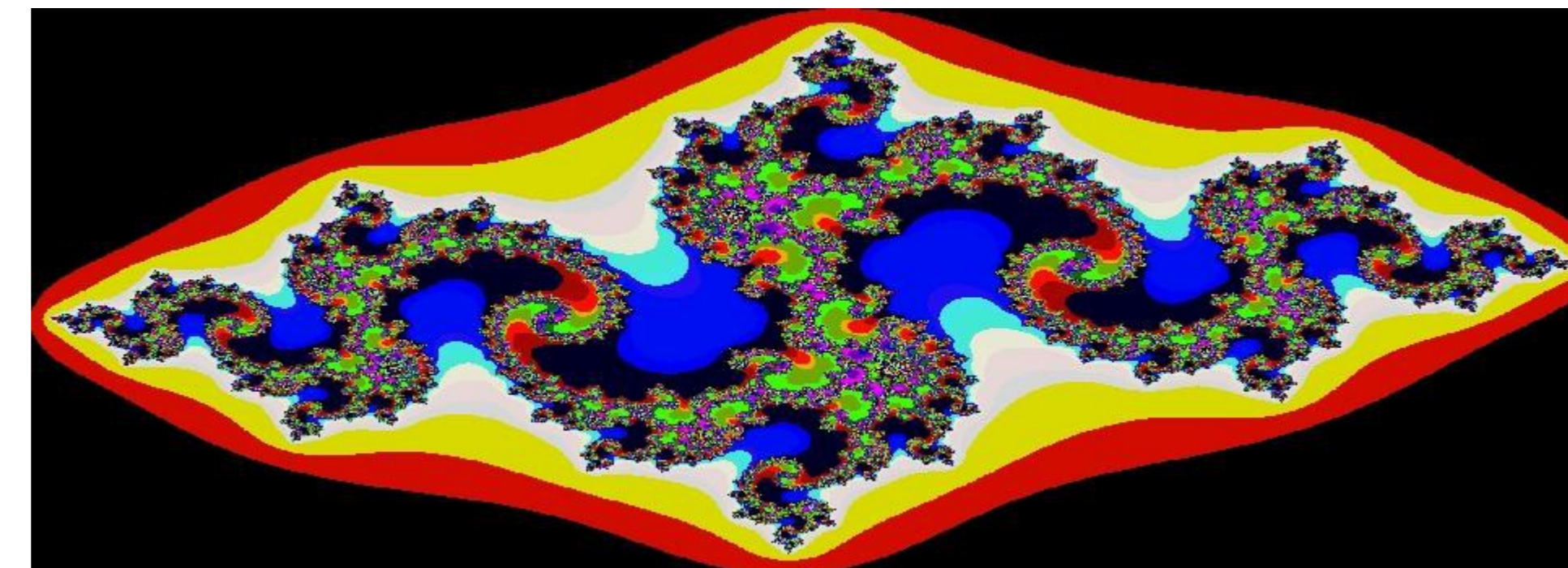
Parallel Algorithm

The initial approach to designing a parallel program which generates the geometric points for a function that make up a Julia set was fairly simple: the total number of data blocks was taken and equally divided between the processors. This approach resulted in poor work division between the processors because some had more points to compute while others less. The improved solution came in the form of the master-slave paradigm. [4] Processor 0 was assigned the role of master, while the remaining processors were slaves. The image was split row by row, with the master processor sending rows to the slave processors and waiting for results. Once a result was received, the master sent an additional row to a slave. After sending all computations, the master waits for completion by the slaves. Upon completion, the master sends a message informing each slave that there are no more computations.

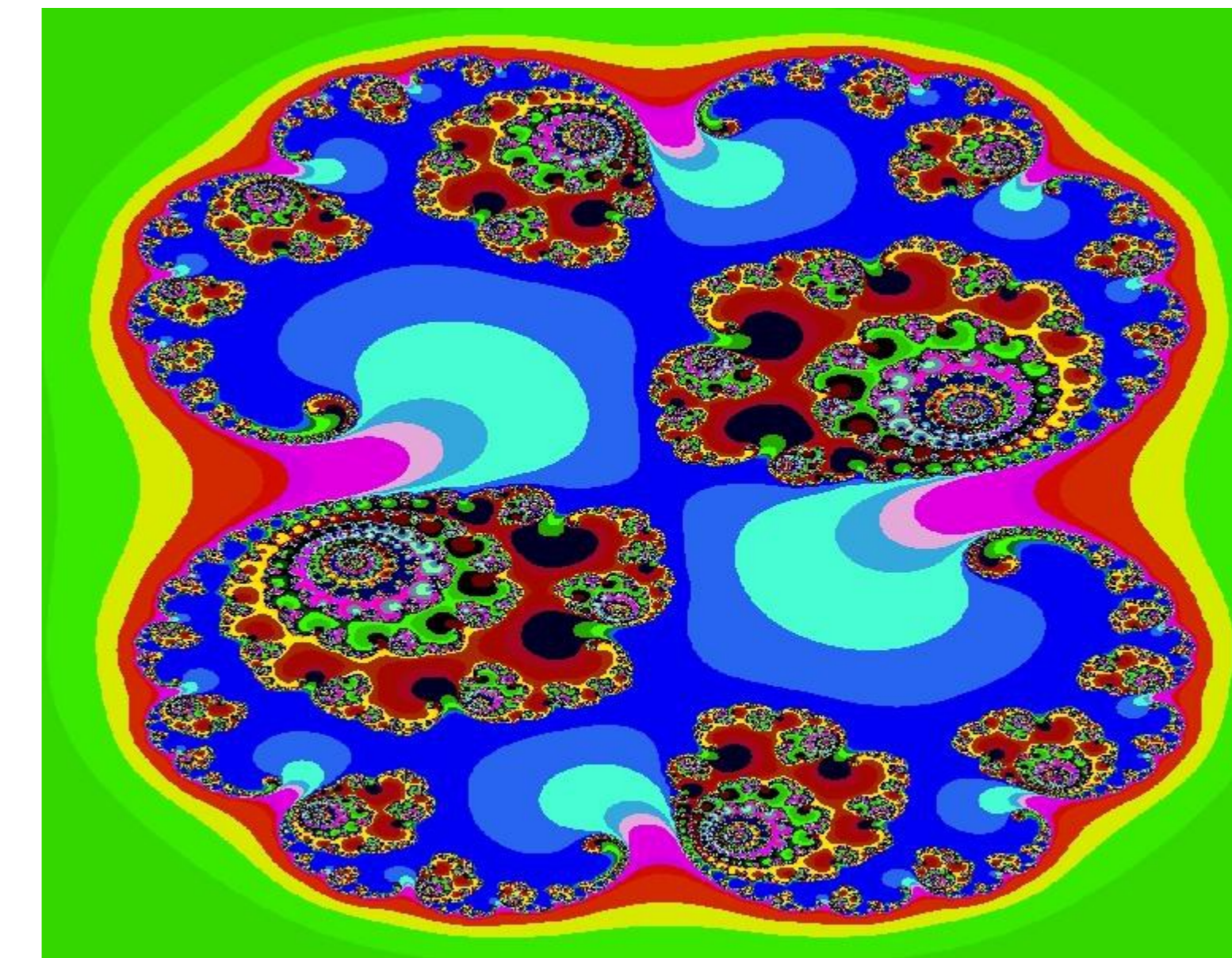
Results

processors	time (s)			
	min	max	max - min	avg
4	257.136136	257.136446	0.000310	257.136318
8	133.481619	133.481899	0.000280	133.481707
16	63.723180	63.724237	0.001057	63.723738
32	31.233345	31.238666	0.005321	31.237287
64	16.230648	16.234519	0.003871	16.233357
128	9.384522	9.587187	0.202665	9.557736

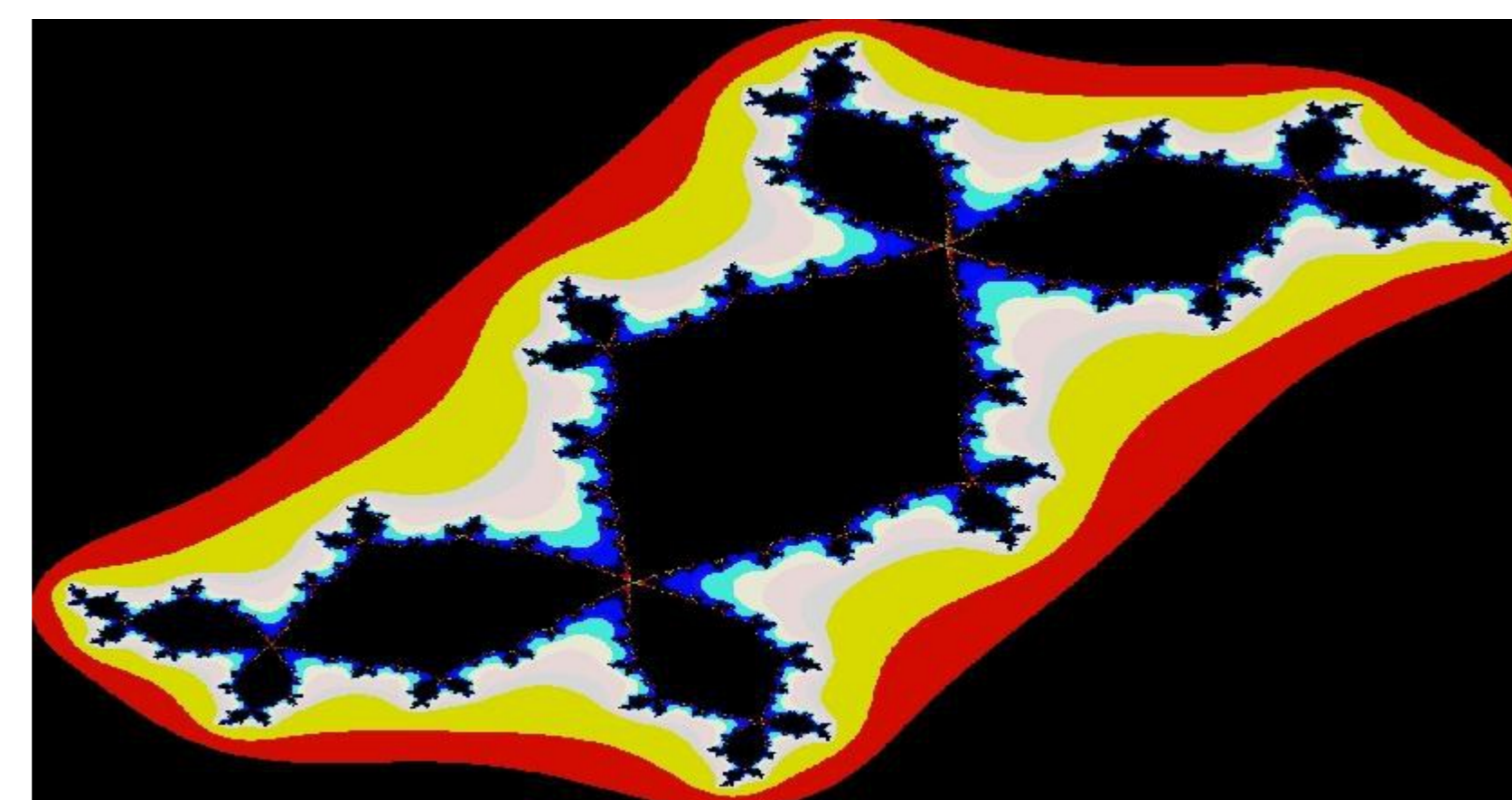
min, *max*, and *avg* represent execution times per processor. *max - min* represents the success of load balancing amongst available processors. Test cases executed on SHARCNET's saw cluster of computers. [5, 6]



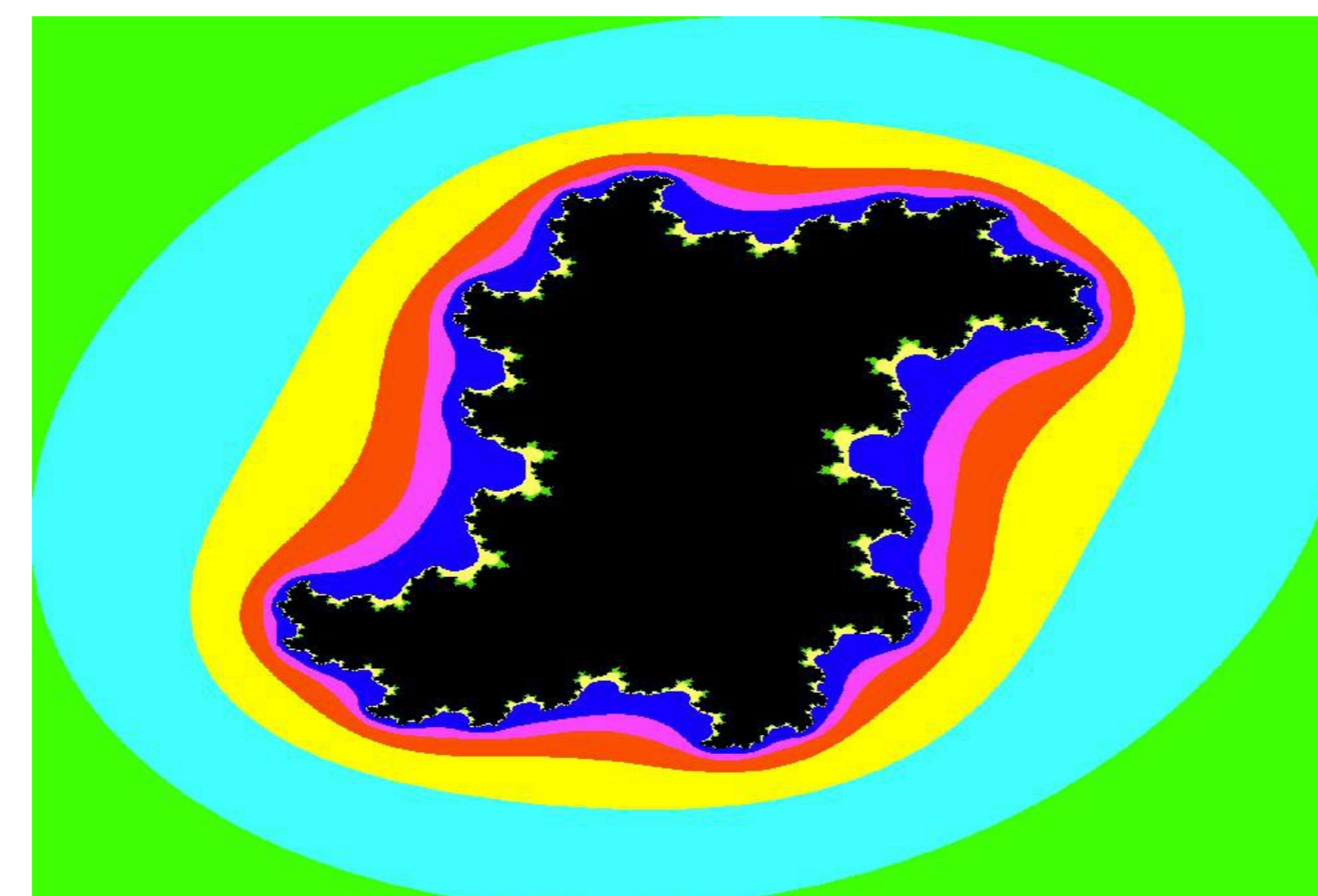
$$Q_c(z) = z^2 - 0.2875 + 0.63508i$$



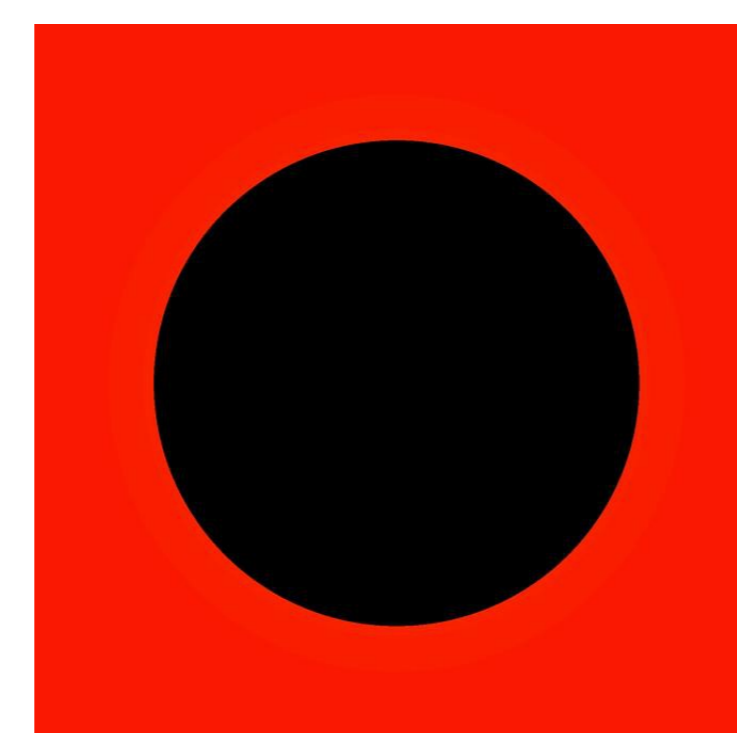
$$Q_c(z) = z^2 - 0.8 - 0.175i$$



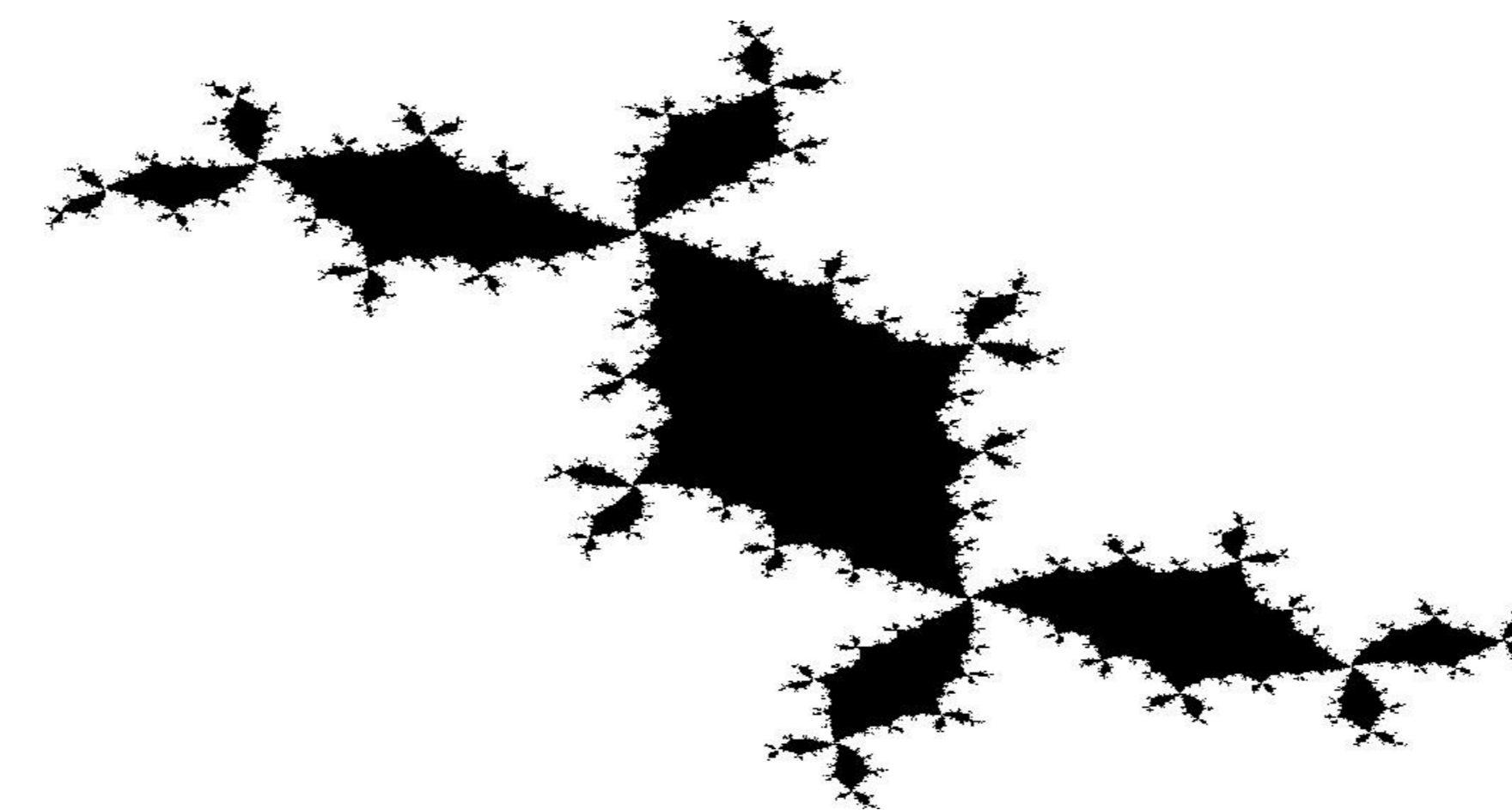
$$Q_c(z) = z^2 - 0.12 - 0.77i$$



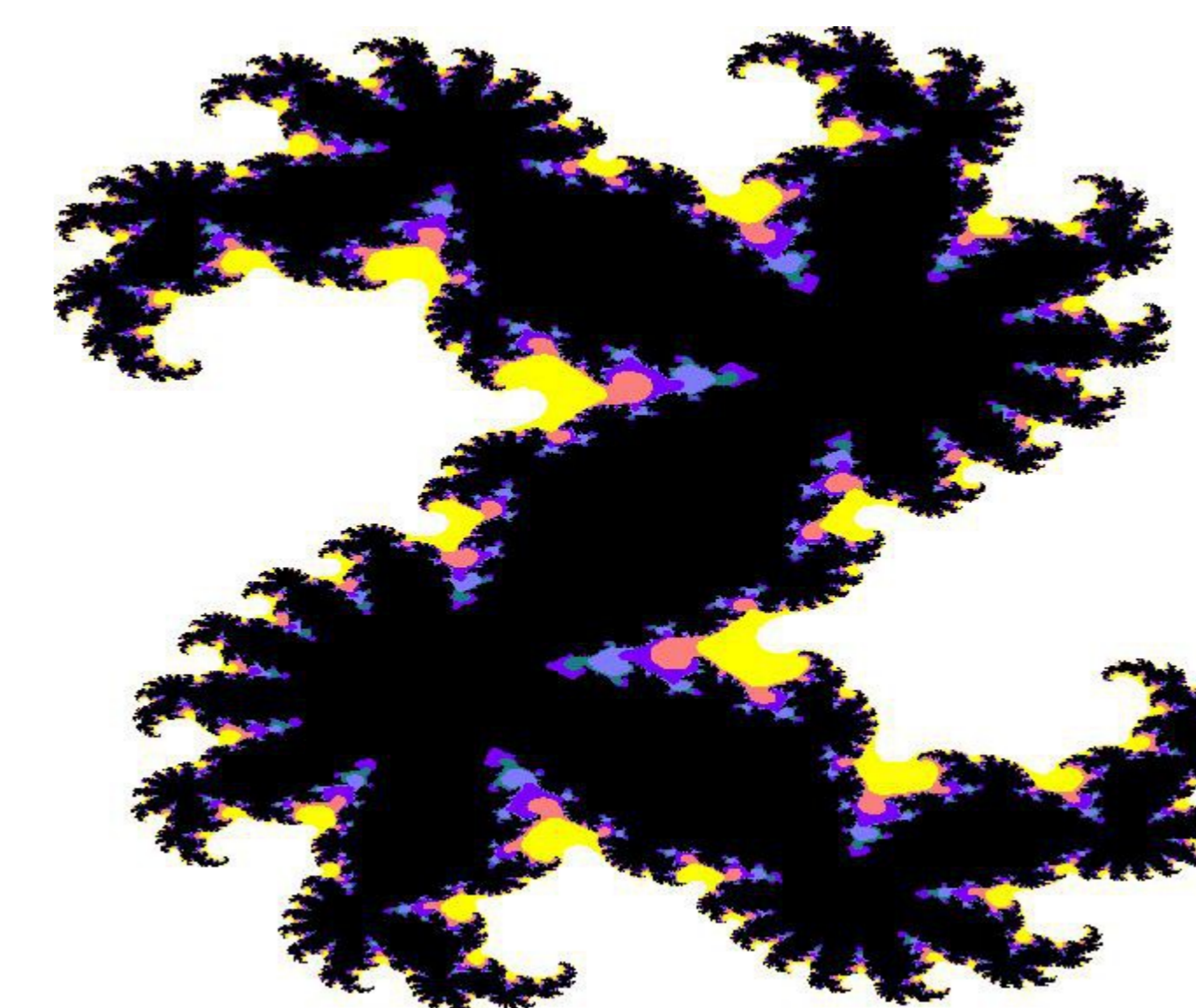
$$Q_c(z) = z^2 + 0.360284 + 0.100376i$$



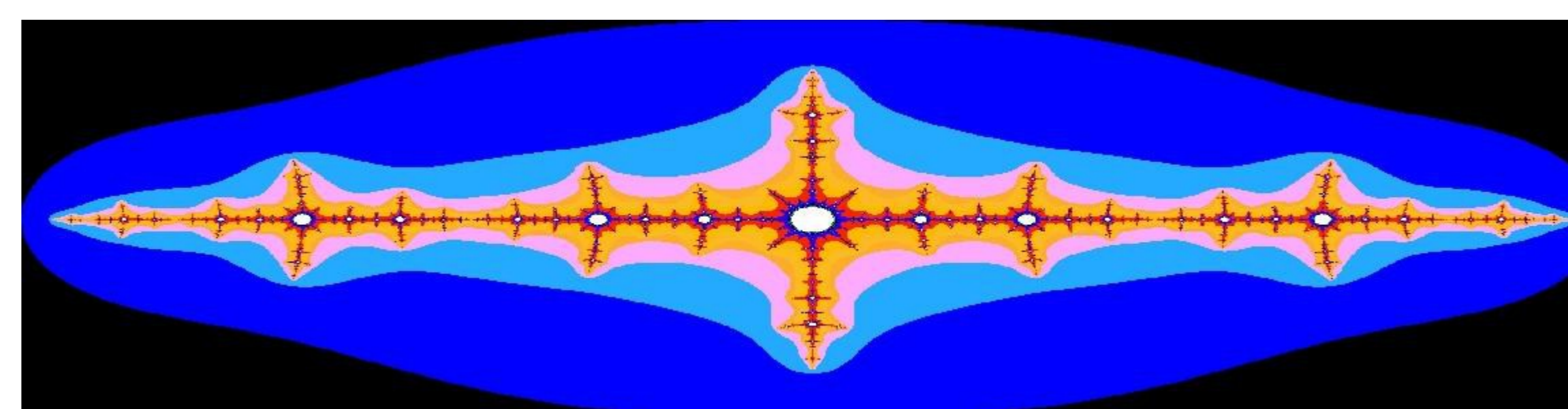
$$Q_c(z) = z^2 + 0$$



$$Q_c(z) = z^2 - 0.1 + 0.8i$$



$$Q_c(z) = z^2 + 0.360284 + 0.100376i$$



$$Q_c(z) = z^2 + 0.285 + 0.01i$$

Conclusion

This project presented two programs to solve the Julia set problem: a parallel program that computes a Julia set and a sequential program which outputs a graphical representation of the Julia set. The parallel program was shown to have done reasonably well in balancing the work load between the available number of processors. Furthermore, parallelism was justified with test results showing that the running time was approximately halved by doubling the number of processors. Overall the project was a success. A problem that benefits from parallel computing was solved and shown to be successful. However there are some interesting topics that may be completed as an extension of this project. The detail of the images may be improved with the inclusion of more colours and output high definition images. In doing so, the topic of fractals may be studied in the resulting high definition images. Another future topic of study may be additional sets, such as the Fatou set and Cantor set.

References

- [1] Devaney, Robert L. 1990. "Chaos, Fractals, and Dynamics: Computer Experiments in Mathematics." Addison-Wesley.
- [2] Julia, Gaston M. 1918. "Memoire sur l'iteration des fonctions rationnelles." Journal de mathematiques pures et appliquees (8e serie, tome 1)
- [3] Weisstein, Eric W. "Julia set." MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/JuliaSet.html>.
- [4] "Parallel Languages/Paradigms: The Master-Slave Paradigm." Department of Computer Science, University of Illinois. <http://charm.cs.uiuc.edu/research/masterSlave>.
- [5] "SHARCNET: Cluster saw.sharcnet.ca." SHARCNET. <https://www.sharcnet.ca/my/systems/show/41>.
- [6] "Saw - Documentation." SHARCNET. <https://www.sharcnet.ca/help/index.php/Saw>.